$\mathcal{F}$-Statistic Search for White Dwarf Binaries in the Third Mock LISA Data Challenge

John T. Whelan[1] **and Reinhard Prix**[2]

[1]Center for Computational Relativity & Gravitation and School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY, USA;
[2]Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Hannover, Germany
john.whelan@astro.rit.edu, reinhard.prix@aei.mpg.de

Abstract

We have performed an analysis of data in the third Mock LISA Data Challenge (MLDC3), using the open-source LALApps $\mathcal{F}$-statistic code to search for simulated LISA signals from galactic white-dwarf binaries (WDBs). Our search pipelines have now been extended to handle WDB frequency evolution, a feature not present in previous MLDCs. We recover amplitude parameters accurately in a targeted search. Our search over the full 4D Doppler parameter space finds many signals, but false alarm rates and parameter recovery are not as good as in our MLDC2 search.

$\mathcal{F}$-Statistic Method

A white-dwarf binary GW signal $s(t)$ is characterized by its *Doppler parameters* θ, i.e. frequency f, frequency derivative f_t and sky-position (ecliptic latitude β, longitude λ), and its *amplitude parameters* $\{\mathcal{A}^\mu\}_{\mu=1}^4 = \mathcal{A}^\mu(h_0, \cos\iota, \psi, \phi_0)$, and can be written as

$$s(t;\mathcal{A},\theta) = \mathcal{A}^\mu\, h_\mu(t;\theta)\,. \quad (1)$$

Maximizing the **likelihood ratio** statistic over the four amplitudes $\mathcal{A}^\mu$ results in maximum-likelihood estimators

$$\mathcal{A}^\mu_{\rm cand}(x;\theta) = \mathcal{M}^{\mu\nu}\,(x\|h_\nu)\;, \quad (2)$$

where $\mathcal{M}^{\mu\nu}$ is the matrix inverse of $\mathcal{M}_{\mu\nu} \equiv (h_\mu\|h_\nu)$. Substituting the amplitude-estimator $\mathcal{A}^\mu_{\rm cand}$ into the likelihood ratio, we obtain the $\mathcal{F}$-statistic:

$$2\mathcal{F}(x;\theta) \equiv |\mathcal{A}_{\rm cand}|^2 \equiv \mathcal{A}^\mu_{\rm cand}\,\mathcal{M}_{\mu\nu}\,\mathcal{A}^\nu_{\rm cand}\,, \quad (3)$$

and so we only need to search over the Doppler-space $\theta = \{f, f_t, \beta, \lambda\}$. If exactly targeting a signal, the expectation value of $2\mathcal{F}$ is $E\left[2\mathcal{F}(x;\theta_{\rm key})\right] = 4 + |\mathcal{A}_{\rm key}|^2$.

Targeted Search

Challenge 3.1 of MLDC3 includes 20 **Verification Binaries** w/known Doppler parameters θ. As in MLDC1[1, 2] and MLDC2[3] we can calculate the $\mathcal{F}$-stat for those parameters (now including $f_t = df/dt$) and deduce the amplitude parameters $\mathcal{A}^\mu$. We illustrate this using the **Training data**, for which the true amplitude parameters were published in advance. We find errors consistent with the expected statistical errors due to noise (Fig.1), even at higher frequencies.

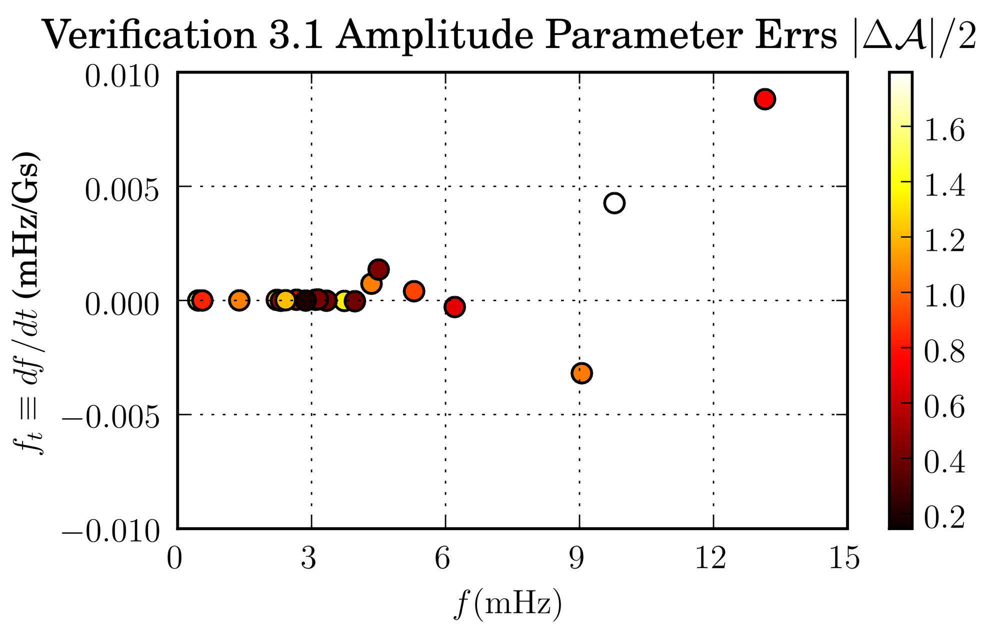

Figure 1: *Recovery of amplitude parameters for verification binaries in training data set, as a function of two of the four Doppler parameters. The quantity plotted on the color scale, $|\Delta\mathcal{A}|/2$, should have an RMS value of unity for Gaussian statistical errors.*

This can also be illustrated by using the inverse of the Fisher matrix $\mathcal{M}_{\mu\nu}$ to determine the error bars on the $\mathcal{A}^\mu$, e.g., $\sigma_{\mathcal{A}^1} = \sqrt{\mathcal{M}^{11}}$. The 20 verification binaries give 80 errors $\mathcal{A}^\mu/\sigma_{\mathcal{A}^\mu}$ which should be Gaussian distributed with zero mean and unit variance. (Fig.2)

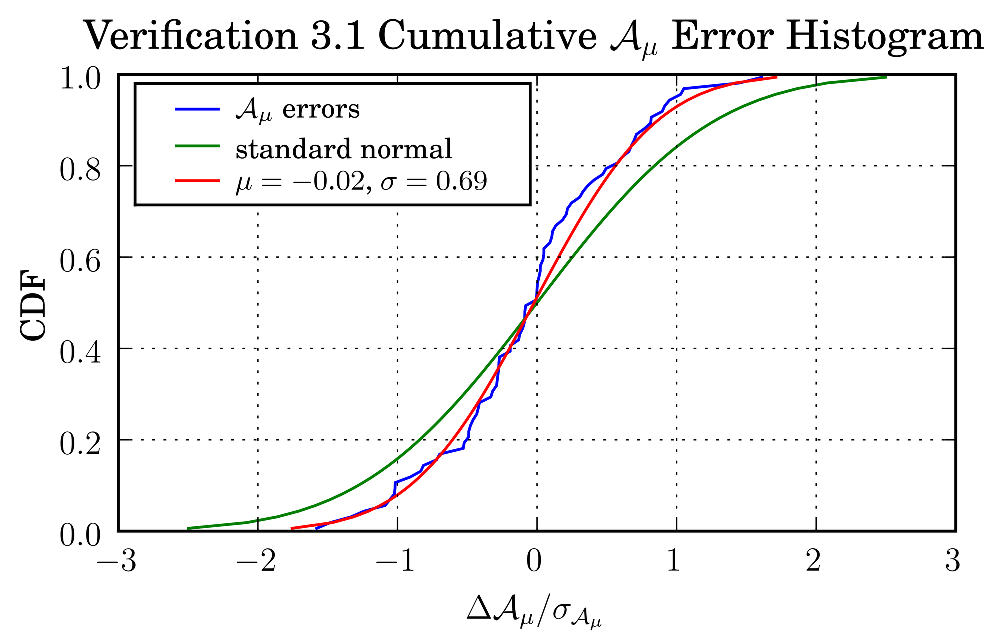

Figure 2: *Distribution of errors in verification binary amplitude parameters relative to Fisher matrix error bars. The errors are consistent with statistical fluctuations.*

Multiple Sources

In addition to verification binaries, challenge 3.1 contains 60 million galactic WD binaries, whose orbital frequency can increase or decrease due to evolution resulting from GW emission or mass transfer. Of those, 40628 were designated as "bright" sources, the norms of whose amplitude parameter vectors are shown in Fig. 3.

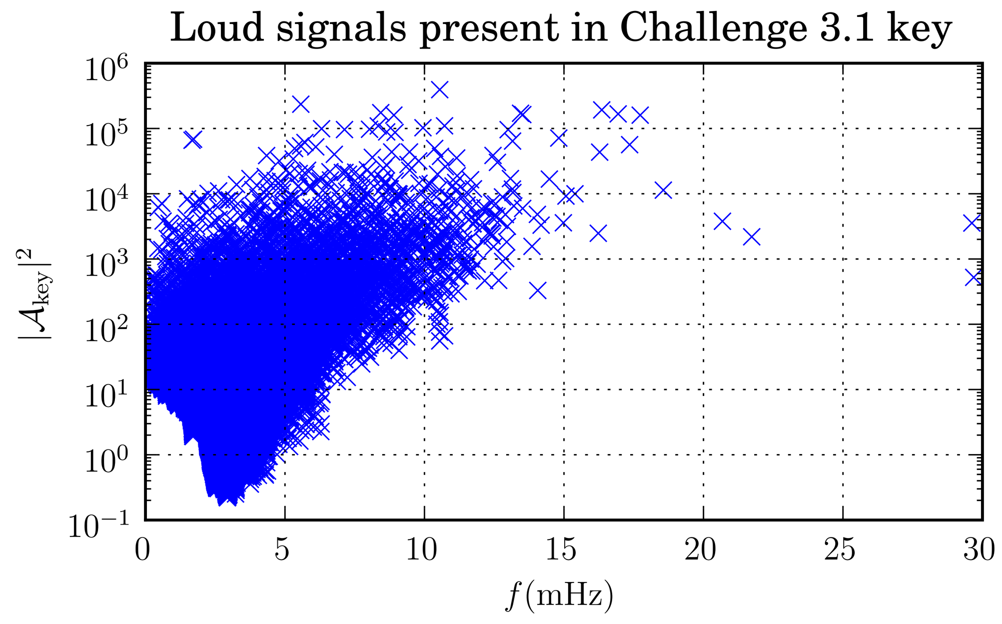

Figure 3: *Sources in Challenge 3.1 "loudest" key file: norms $|\mathcal{A}_{\rm key}|^2$ (corresponding to ${\rm SNR}^2$) of injected amplitude-vectors as function of frequency f.*

The large number of detectable sources makes it difficult to distinguish the actual ("primary") from "secondary" maxima of the detection statistic $2\mathcal{F}(x;\theta)$ in Doppler parameter space. Our pipeline is based on the empirical observation that primary maxima show better coïncidence between different TDI variables X, Y, Z than secondary maxima. The coïncidence criterion is based on the **metric** g_{ij} in Doppler parameter space, namely

$$m = g_{ij}\, d\theta^i\, d\theta^j + \mathcal{O}(d\theta^3)\,, \quad (4)$$

which attributes a "distance" m to the Doppler offsets $d\theta = \{df, df_t, d\beta, d\lambda\}$.

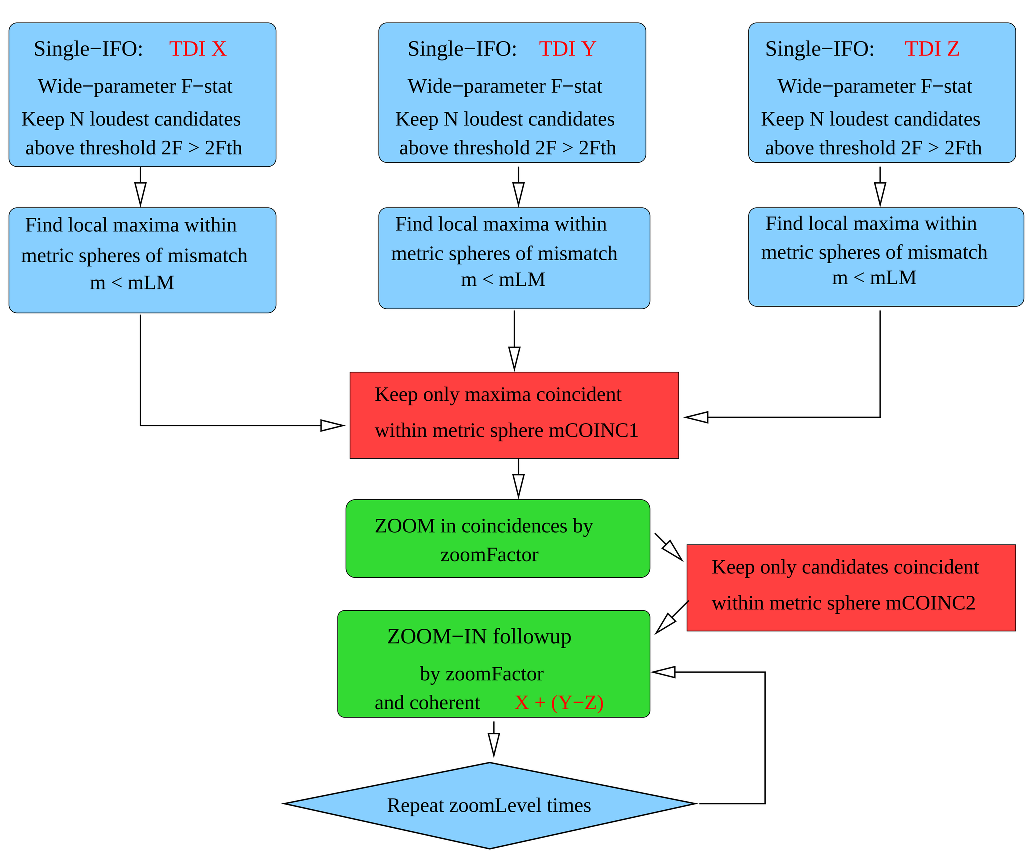

Figure 4: *Hierarchical coïncidence pipeline. True signals are identified by looking for coïncidences between single-observable searches, after which their parameters are refined using a full multi-detector search.*

Blind Search Results

For our MLDC3 entry, we performed a search for signals across the **4D** Doppler parameter space. The search was only conducted up to 16 mHz because so few signals were present between 16 and 30 mHz. We initially attempted to set the resolution in f_t using the projected metric perpendicular to the f direction, but this led to systematic errors and biases in f and f_t and corresponding large amplitude parameter errors in our preliminary results.[4] This problem has been solved by using the unprojected metric to set the initial f_t resolution.

Freqs	Found	Missed	False
0–4 mHz	644	2730	16
4–8 mHz	865	1992	69
8–12 mHz	268	217	31
12–16 mHz	39	5	7
Total	1816	4944	123

Table 1: *Signals found in Challenge 3.1, along with missed sources and false alarms. Only sources with $|\mathcal{A}_{\rm key}|^2 > 40$ are included in the "missed" category.*

Our pipeline identified 1939 candidate signals in the Challenge 3.1 data, with $2\mathcal{F}$ values ranging from 50.7 to 3.43×10^5. Of the 40628 "bright" sources in the key, 6760 had $|\mathcal{A}_{\rm key}|^2 > 40$ and $f < 16\,\rm mHz$. To evaluate our results, we identified each candidate with the loudest "bright" source within a Doppler mismatch of $m \leq 1$. If there was no "bright" source within that Doppler window, we considered the candidate to be a false alarm. The results of this identification are summarized in Table 1. This search recovers a comparable number of signals to our MLDC2 search[3], but with a higher false alarm rate. The amplitude parameters are also recovered with larger errors relative to the verification binary search:

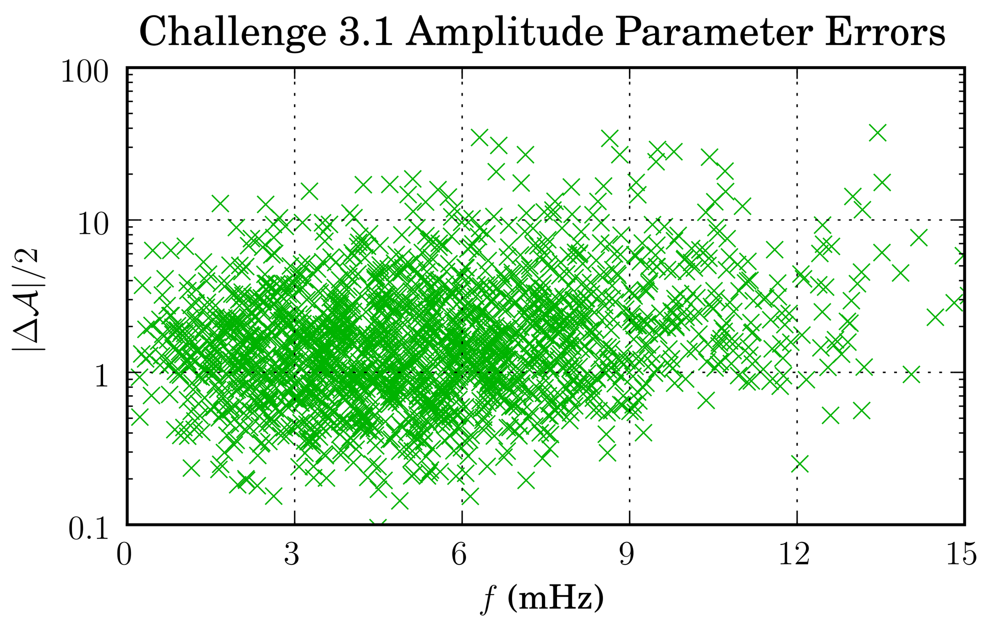

Figure 5: *Amplitude parameter errors for Challenge 3.1. The quantity plotted, $|\Delta\mathcal{A}|/2$, would have an RMS value of 1 due to Gaussian statistical error.*

Quantitatively, we see that the amplitude parameter errors are about twice as large as the Fisher matrix estimates. This is larger than in the MLDC3 verification binary search and in the MLDC2 search[3], but smaller than in our preliminary results[4]

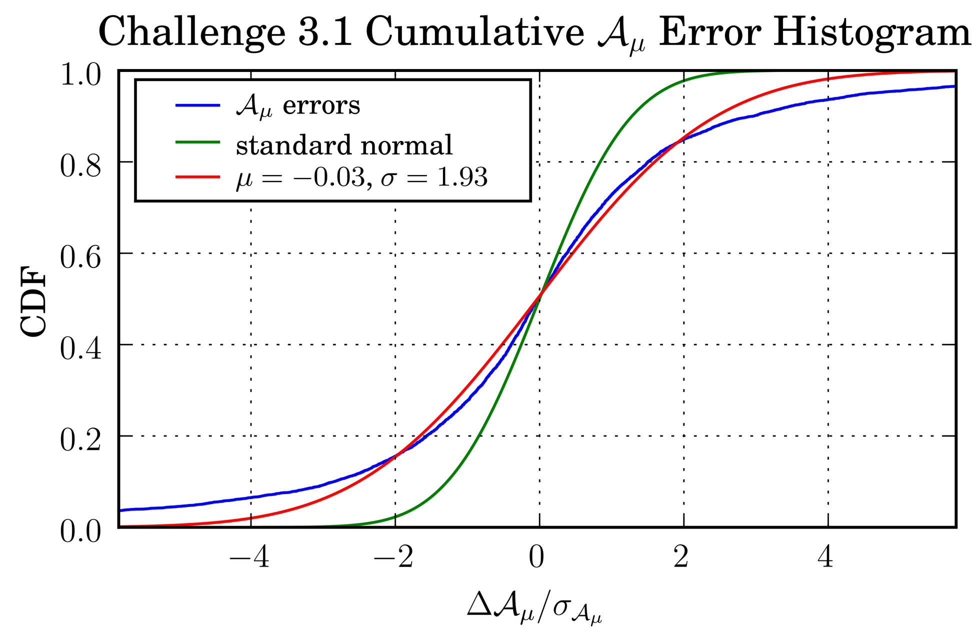

Figure 6: *Distribution of errors in amplitude parameters relative to Fisher matrix error bars. The errors are somewhat larger than would be expected due to statistical errors from noise.*

The Doppler parameter errors are also slightly larger than statistical expectations, but the large systematic errors & biases seen in our preliminary search[4] have been removed by the finer initial f_t resolution.

	$\Delta f/\sigma_f$	$\Delta f_t/\sigma_{f_t}$	$\Delta\beta/\sigma_\beta$	$\Delta\lambda/\sigma_\lambda$	$\Delta\mathcal{A}^\mu/\sigma_{\mathcal{A}^\mu}$
mean	0.01	0.0001	−0.0004	−0.003	−0.03
std dev	1.25	1.26	0.95	0.97	1.93

Table 2: *Parameter errors in Challenge 3.1. The sky position accuracy is consistent with statistical fluctuation, while the frequency and its derivative have slightly larger errors than theoretically expected.*

Summary

We have extended our pipeline from previous MLDCs to handle the WDBs with **frequency evolution** in MLDC3. The program can recover amplitude parameters of verification binaries. The multiple-signal pipeline recovers $\mathcal{O}(2000)$ signals, with f, f_t and amplitude parameter errors slightly larger than theoretical expectations, and has a higher false alarm rate than our MLDC2 search.[3]

References

[1] Prix & Whelan CQG**24**, S565 (2007)
[2] Whelan, Prix & Khurana CQG**25**, 184029 (2008)
[3] Whelan, Prix & Khurana, LIGO-P080087-Z
[4] Whelan & Prix, LIGO-G090018-Z