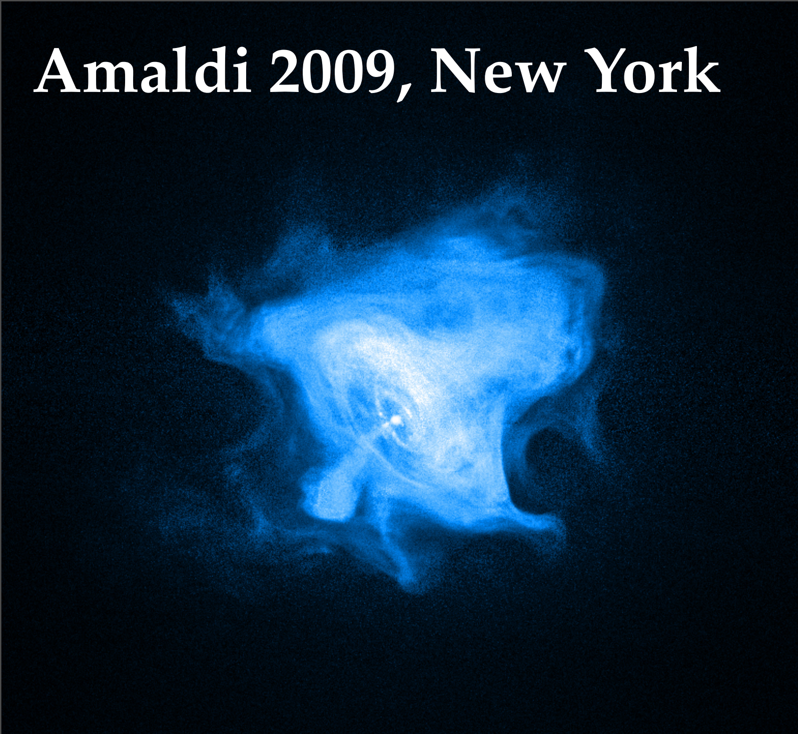
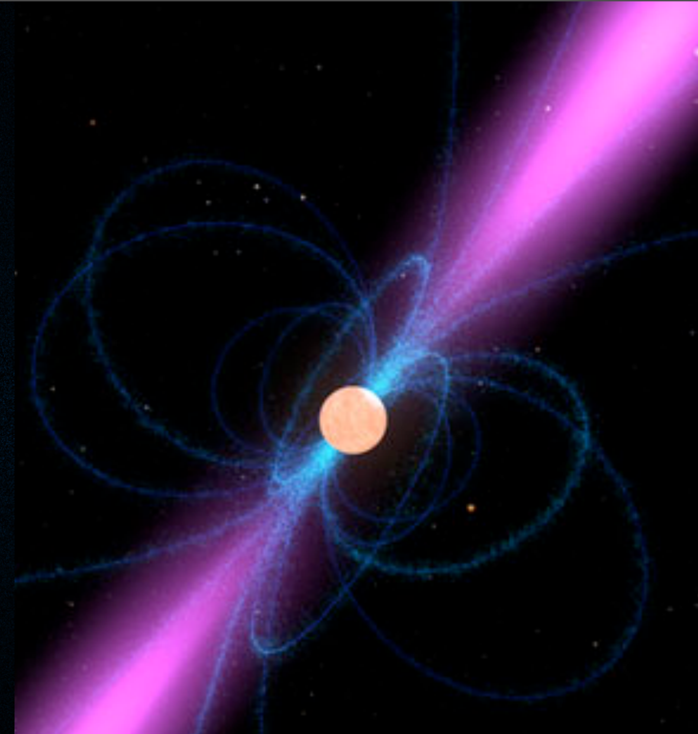
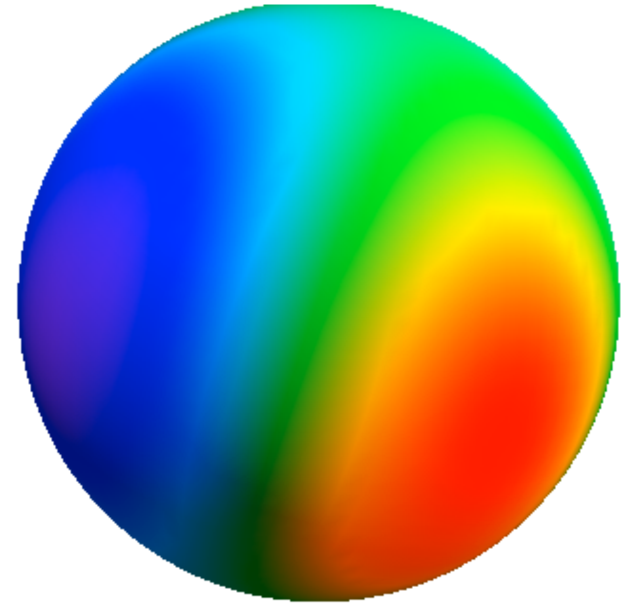
Amaldi 2009, New York







# A Bayesian Search For Gravitational Wave Ring-downs Associated With Pulsar Glitches

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LIGO-G0900574-v1

# Outline

- Pulsar Glitches & Gravitational wave ring-downs
- Bayesian Inference
- A Bayesian GW Search
- A Glitch In PSR B0833-45
- Interpreting Upper Limits

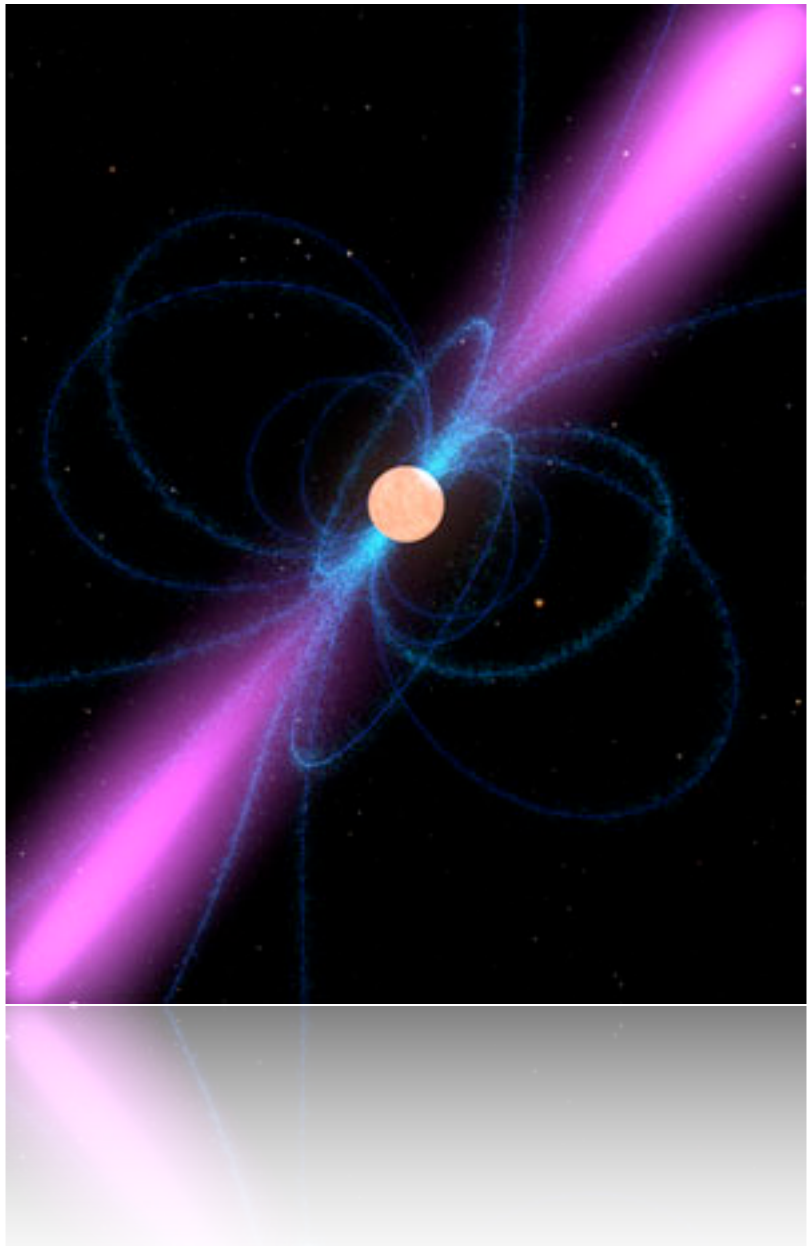


Image: pulsar (Credit: NASA)

# Pulsar glitches

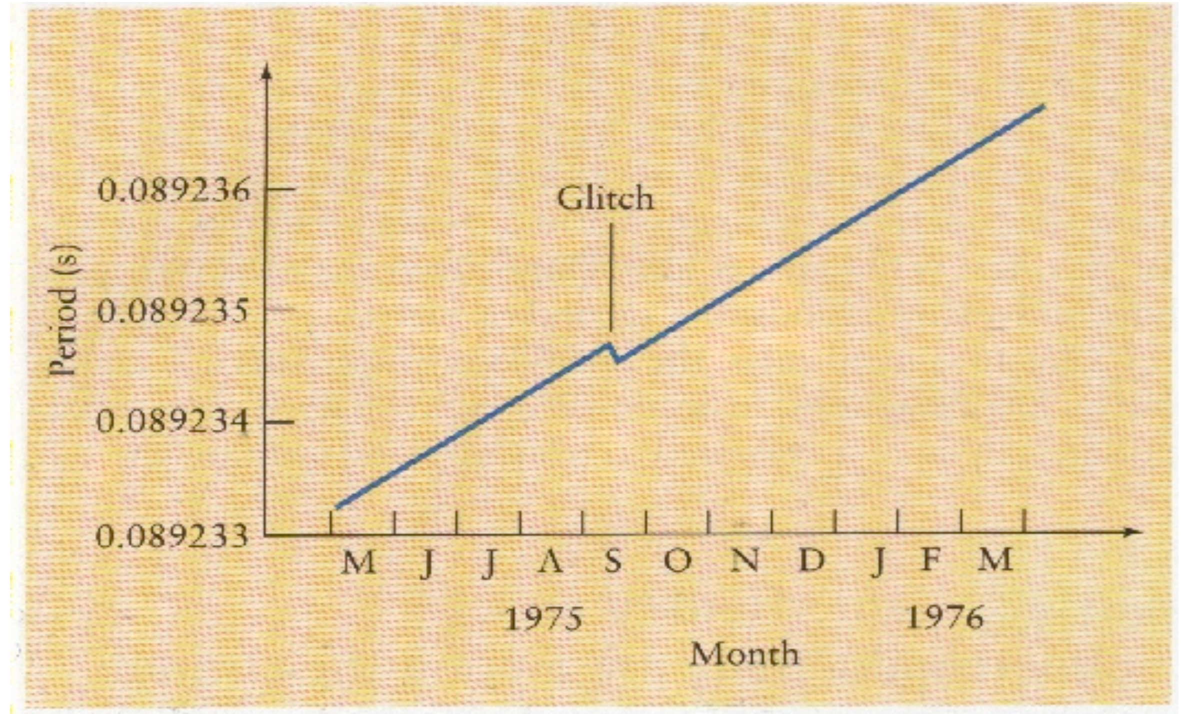
- Observe sudden step increase in rotation rate
- At some critical lag frequency $\Omega_{lag}$, interior super-fluid couples to the crust, imparting angular momentum & energy:

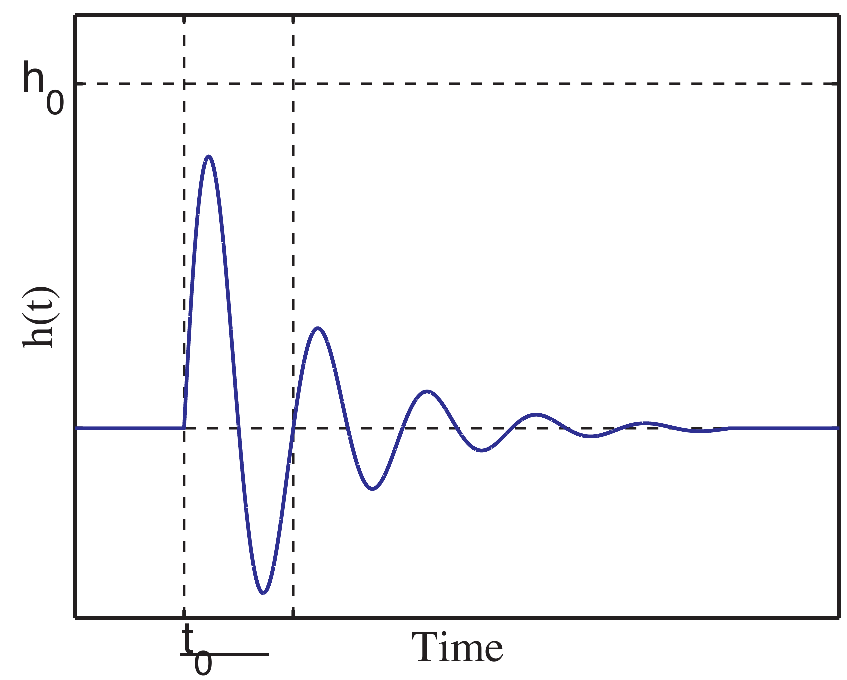
$$\Delta J \sim I_* \Delta\Omega \qquad \Delta E = \Delta J \Omega_{\mathrm{lag}}$$

- Large glitches: $\Delta\Omega/\Omega \sim 10^{-6}$ so

$$\Delta E \sim 10^{-13}\text{-}10^{-11} \mathrm{M}_\odot c^2$$

- Possible that this sudden jolt in the rotation could excite oscillations
- Various oscillatory modes exist (f-modes, p-modes, w-modes)
- Gravitational wave emission damps non-axisymmetric oscillations
- Mode frequencies determined by equation of state




$$h_0 \sim 10^{-24} \left(\frac{E_{\mathrm{GW}}}{10^{-11} M_\odot c^2}\right)^{1/2} \left(\frac{15\,\mathrm{kpc}}{D}\right) \left(\frac{2\,\mathrm{kHz}}{\nu_f}\right) \left(\frac{200\,\mathrm{ms}}{\tau_f}\right)^{1/2}$$

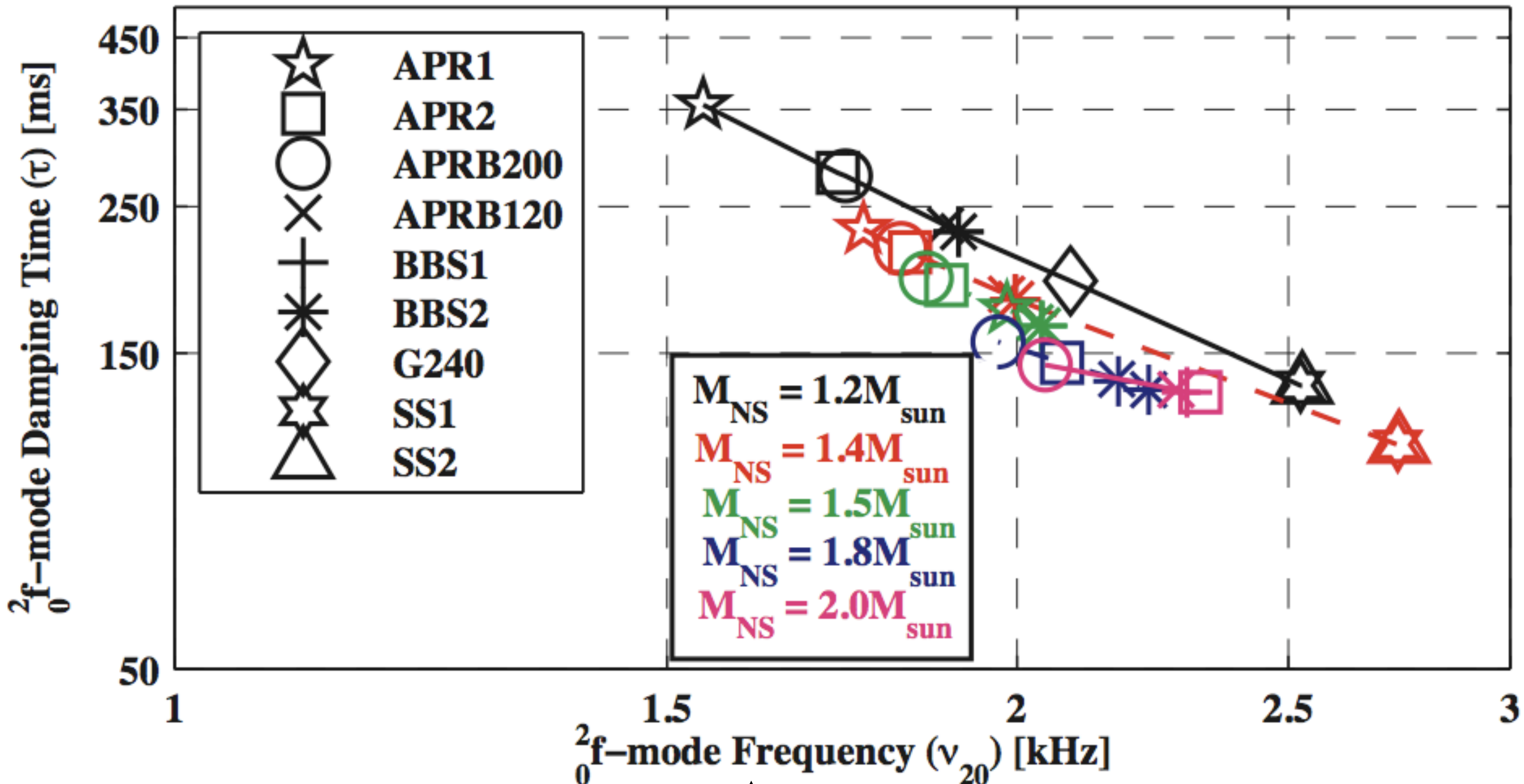
# Neutron Star QNM Parameter space

- *f*-mode frequencies and damping times
- symbol shape = EOS
- Colour = NS mass

- Figure created from data in Benhar et al (2005) - recent EOS calculations and representative but not exhaustive

- Adopt flat priors on signal frequency $f_0$ and decay time $\tau$:

$\tau^{(upp)} = 0.5$ s, $\tau^{(low)} = 0.05$ s




$$p(\tau|\mathrm{M}_1, I) = \frac{1}{\tau^{(\mathrm{upp})} - \tau^{(\mathrm{low})}}$$

$f_0^{(upp)} = 3$ kHz, $f_0^{(low)} = 1$ kHz

$$p(f_0|\mathrm{M}_1, I) = \frac{1}{f_0^{(\mathrm{upp})} - f_0^{(\mathrm{low})}}$$

# Bayesian Inference

## Bayes' Theorem:

$$\Pr(\theta_0|d, I) = \frac{\Pr(\theta_0|I) \times \Pr(d|\theta_0, I)}{\Pr(d|I)}$$

## Evidence:

$$\Pr(d|I) = \sum_{k=1}^{N} \Pr(\theta_k, d|I)$$

$$= \sum_{k=1}^{N} \Pr(\theta_k|I)\Pr(d|\theta_k, I)$$

- **Evidence**: likelihood, marginalised over all model parameters (aka "marginal likelihood", "global likelihood")

- Suppose we have 2 models $M_1$ and $M_2$. Form the "odds ratio" $O_{12}$

$$O_{12} = \frac{\Pr(M_1|d, I)}{\Pr(M_2|d, I)} = \frac{\Pr(M_1|I)}{\Pr(M_2|I)} \times \frac{\Pr(d|M_1, I)}{\Pr(d|M_2, I)}$$

- The **prior odds** express initial bias for $M_1$ over $M_2$
- The **Bayes factor** (evidence ratio) expresses the influence of the data and incorporates a quantitative Occam's razor effect through the choice of priors:

*"entia non sunt multiplicanda praeter necessitatem" ~ "all things being equal, the simplest argument is the best"* (William of Occam circa 14th century)

# A Bayesian GW Search (1)

**Use the odds ratio as a detection statistic:**

$$O_{s,n} = \frac{\Pr(M_s|I)}{\Pr(M_n|I)} \frac{\Pr(D|M_s,I)}{\Pr(D|M_n,I)}$$

$$\Pr(D|M_s,I) = \int_\mu p(\vec{\mu}|M_s,I)p(D|\vec{\mu},M_s,I)\,d\vec{\mu}.$$

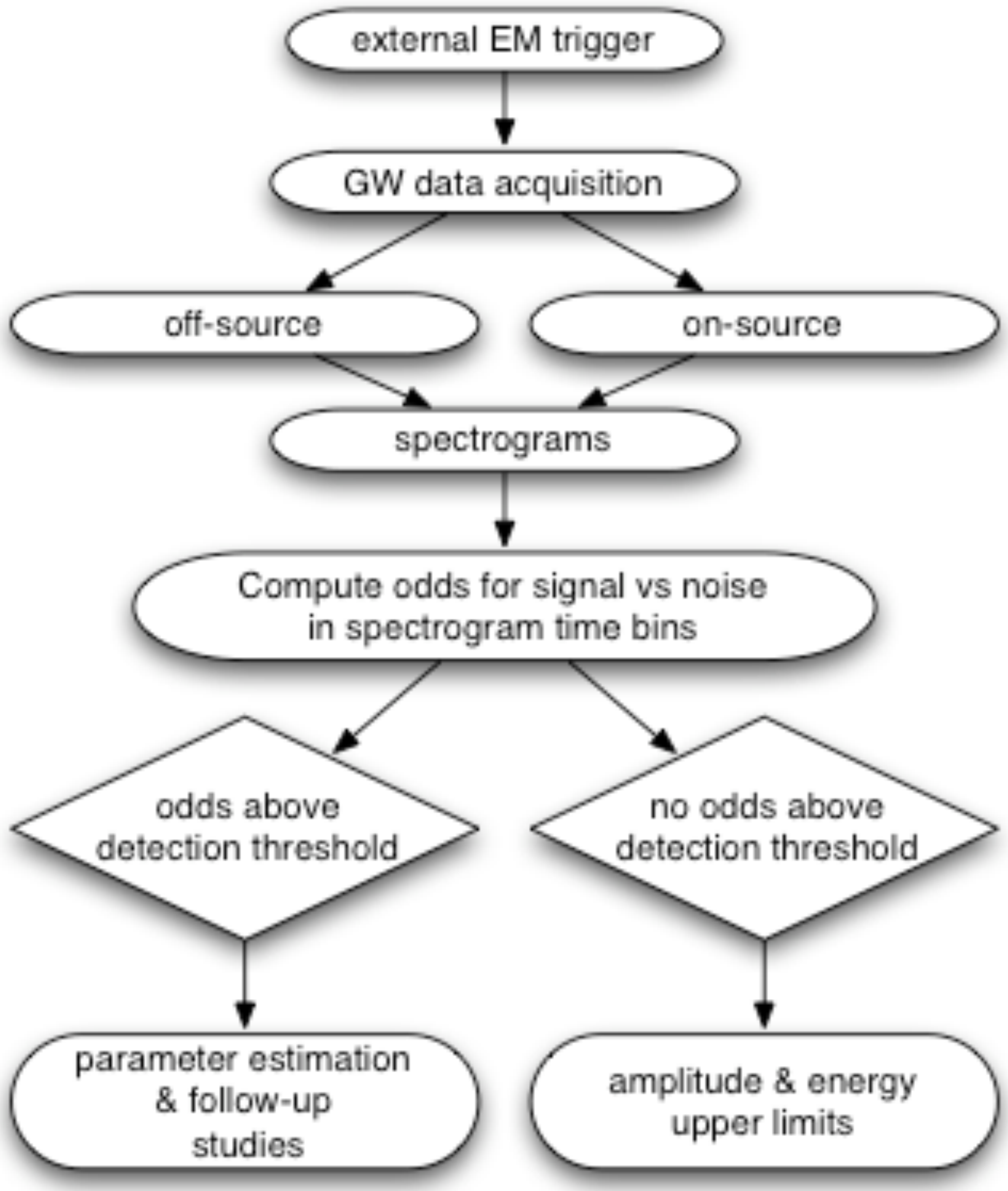
$\vec{\mu} = \{h_0,\ \nu_0,\ \tau,\ t_0\}$ **evidence for signal**

$\Pr(D|M_n,I) =$ **evidence for Gaussian noise**

**Upper limits directly from *marginal* amplitude/ energy posteriors:**

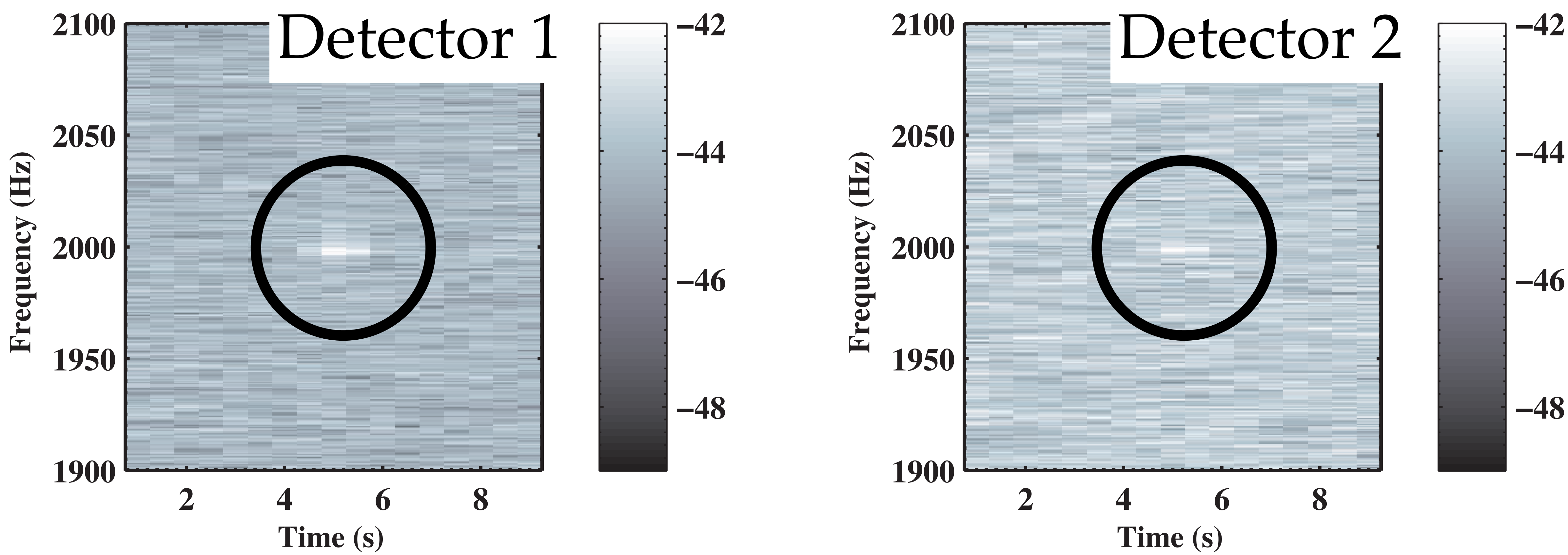
$$0.9 = \int_0^{h_\alpha} p(h_0|D,M_s,I)\,dh_0$$

## Search Pipeline



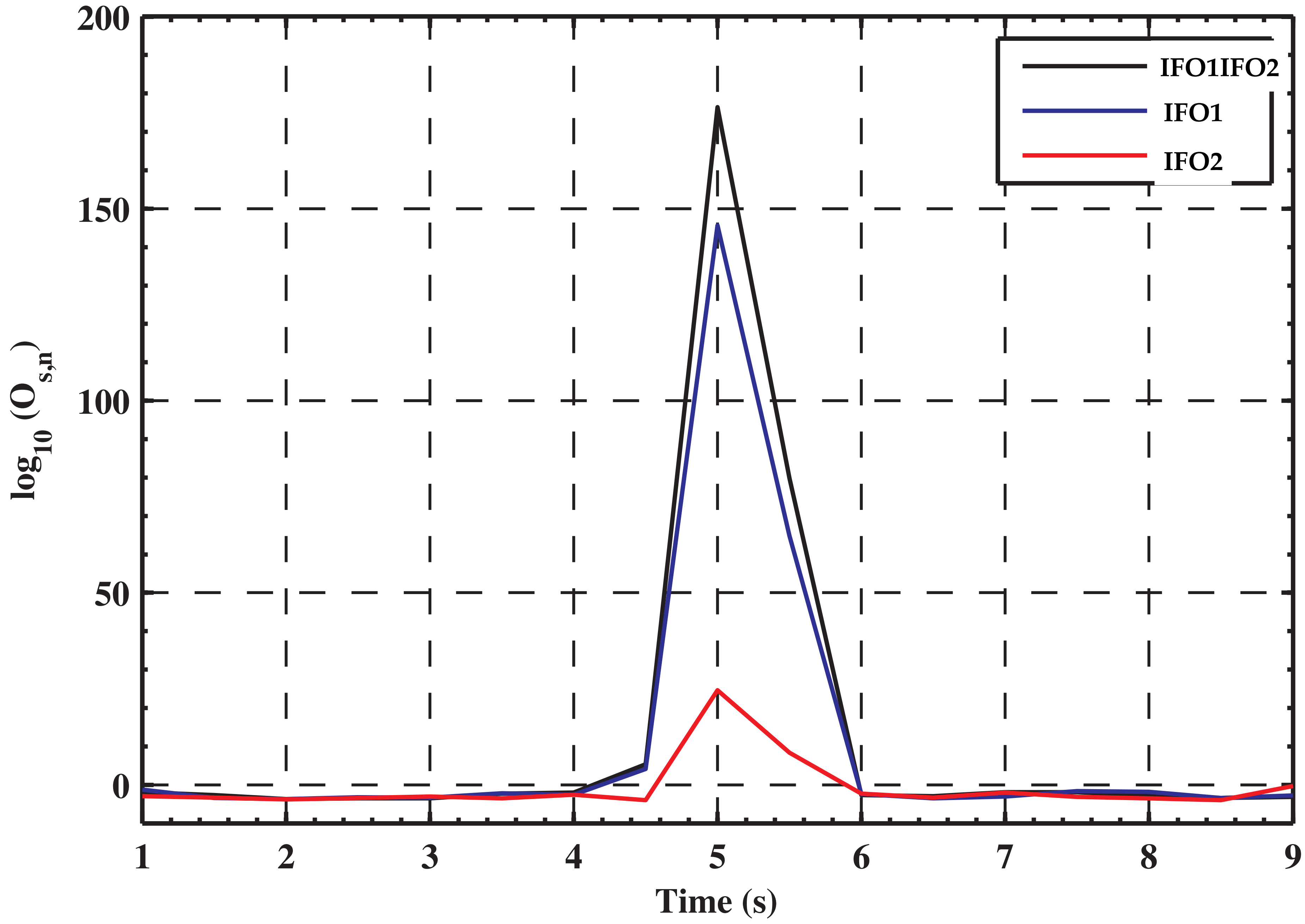
# Detection Demonstration (1)

- Imagine a pulsar glitch is reported at time $T_{glitch}$ with absolute timing uncertainty +/- 5 seconds
- Have coincident data from 2 LIGO detectors in Hanford (i.e., co-located)
- **Simulate** scenario by synthesising 10 seconds of **Gaussian white noise**:
    - Signal injection: $h_0=7\times10^{-21}$, $f_0=2$ kHz, $\tau=0.2$ s, $t_0=5$s
    - Detector 1 (4km instrument) noise spectral density = $1 \times10^{-44}$ $Hz^{-1}$
    - Detector 2 (2km instrument) noise spectral density = $4 \times10^{-44}$ $Hz^{-1}$



# Detection Demonstration (2)

- Odds ratios for signal vs noise in each spectrogram time bin:



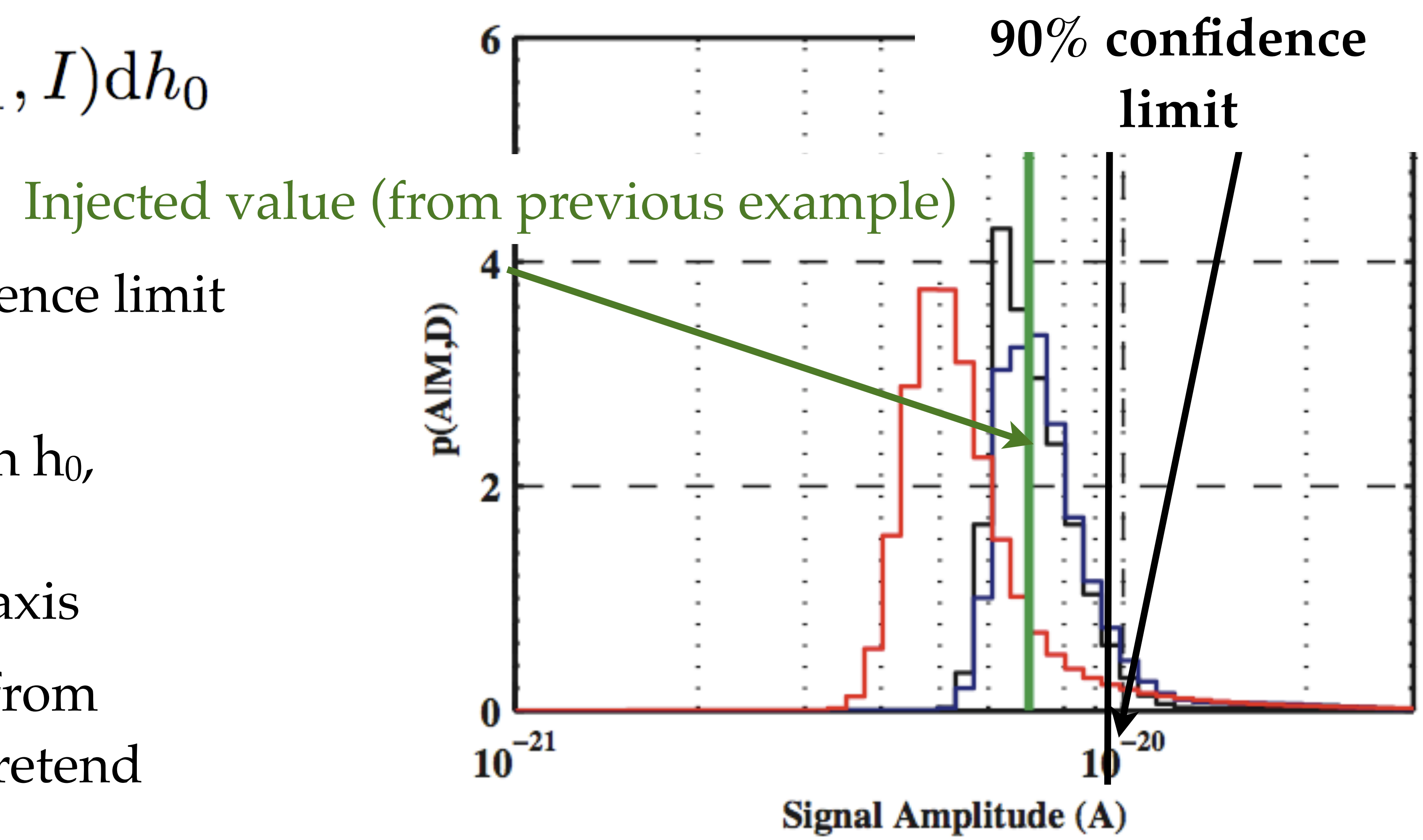

- Signal is seen in both detectors (i.e., odds >> background level)

# Upper Limits On GW Emission

- Detection with initial LIGO unlikely: form direct upper limit on GW emission as ring-down
  - Compute odds ratios in spectrogram time bins
  - Pick time bin with greatest odds ratio
  - GW signal must generate odds <= loudest odds
  - Compute upper limit from marginal posterior on signal amplitude in that time

$$\alpha = \int_0^{h_0} p(h_0|D, \mathrm{M}_1, I)\mathrm{d}h_0$$

  - $\alpha$ = confidence limit
  - Typically use 90% confidence limit so $\alpha = 0.9$
  - Get marginal posterior on $h_0$, integrate up to 0.9, read corresponding value off axis
  - Plot: marginal posterior from previous injection (i.e., pretend this was a loudest event)



# Energy Upper Limits (1)

- Ultimately want to relate inferred $h_0$ to physical parameters of system (c.f., moment of inertia / ellipticity plane in continuous wave searches)
- Transient events here so relate to energy radiated during QNM ringing
- Inferred energy dependent on amplitude, frequency, decay time and source distance:

$$E = \frac{c^3 h_0^2 \omega_0^2 R_0^2 \tau}{4G}$$

- $E$ = GW energy
- $R_0$ = source distance
- $\omega_0$ = angular frequency

- How to get energy posterior? Marginalisation:

$$\begin{aligned} p(E_0|D,M) &= \int_{\underline{\theta}} p(E_0,\underline{\theta}|D,M)\mathrm{d}\underline{\theta} \\ &= \int_{\underline{\theta}} p(E_0|\underline{\theta},D,I)p(\underline{\theta}|D,I)\mathrm{d}\underline{\theta} \end{aligned}$$

- $\underline{\theta} = \{h_0, \omega_0, \tau, R_0\}$
- $E_0$ = some specific value of E

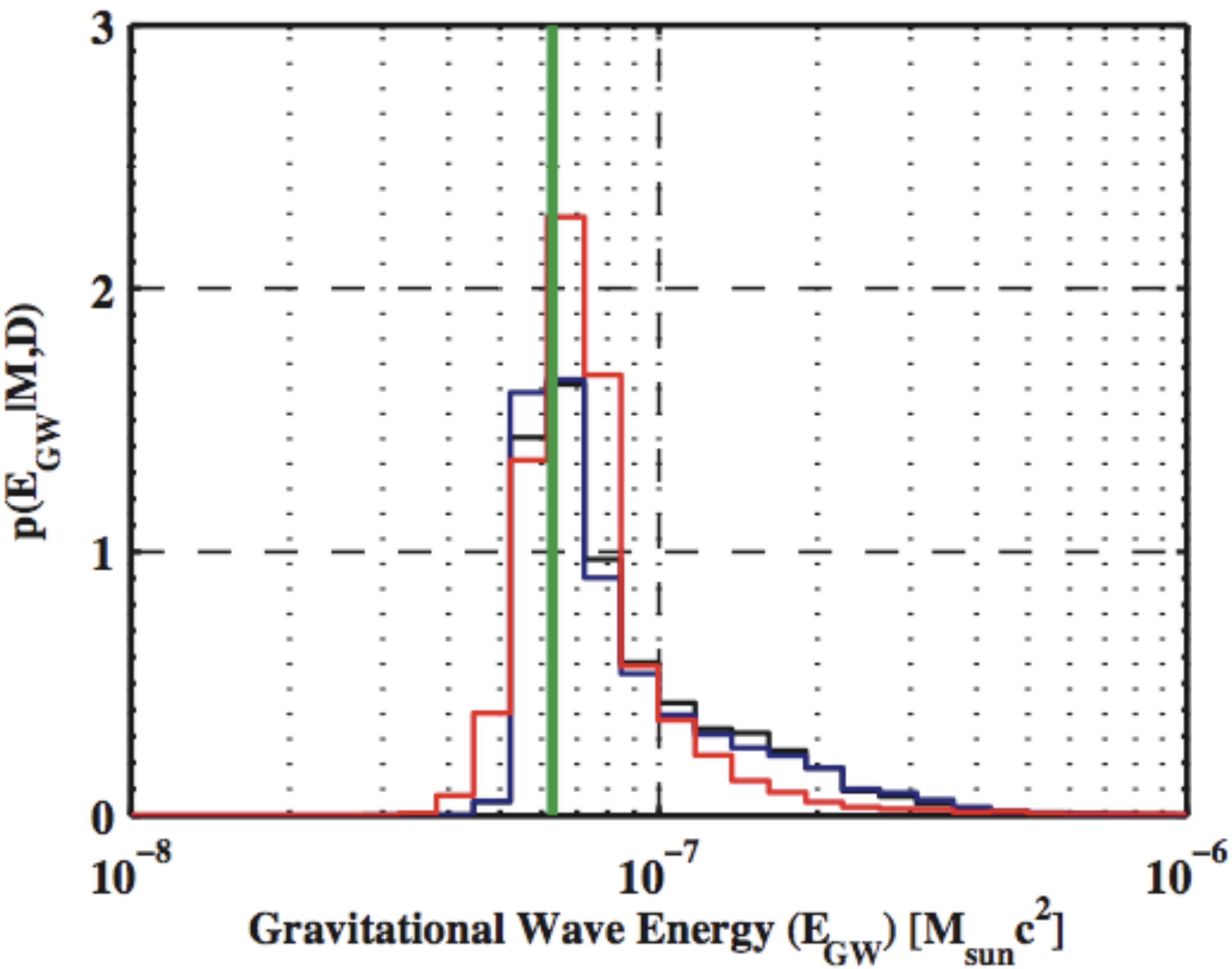
- Knowledge of $E$ is determined entirely by knowledge of $\underline{\theta}$ so immediately write down:

$$p(E_0|\underline{\theta},\mathrm{M}_1,I) = \delta\left[E_0 - E(\underline{\theta})\right]$$

- $E(\underline{\theta})$ = energy given by parameters $\underline{\theta}$

# Energy Upper Limits (2)

- Energy posterior from demonstration example:




- Note: assumed $R_0$=293 pc here
- Could instead take a Gaussian prior on the distance and include marginalisation (with a faster algorithm...like nested sampling)

# A Glitch In PSR B0833-45 (1)

- Prototyped method & pipeline on Vela (PSR B0833-45) glitch during fifth LIGO science

| PSR B0833-45 | | |
|---|---|---|
| Right ascension | $\alpha$ | $08^h35^m20.61149°$ |
| Declination | $\delta$ | $-45°10'34.8751''$ |
| Spin frequency | $\nu_s$ | 11.1946499395 Hz |
| Distance | $R_0$ | $293^{+19}_{-17}$ pc |
| Spin inclination | $\zeta$ | $63.60^{+0.07}_{-0.05} \pm 1.3°$ |
| Position Angle | $\psi$ | $130.63^{+0.05}_{-0.07}$ |
| Glitch epoch (GPS time) | $T_{\mathrm{glitch}}$ | $839457047 \pm 86$ |
| Fractional glitch size | $\Delta\nu/\nu_s$ | $2.620 \times 10^{-6}$ |
| LIGO antenna factors | $F_+, F_\times$ | 0.26, 0.33 |

**Large glitch (largest ~3.1 x10$^{-6}$)**

**Very close!**

**Orientation known**

**Glitch epoch and approximate uncertainty known**

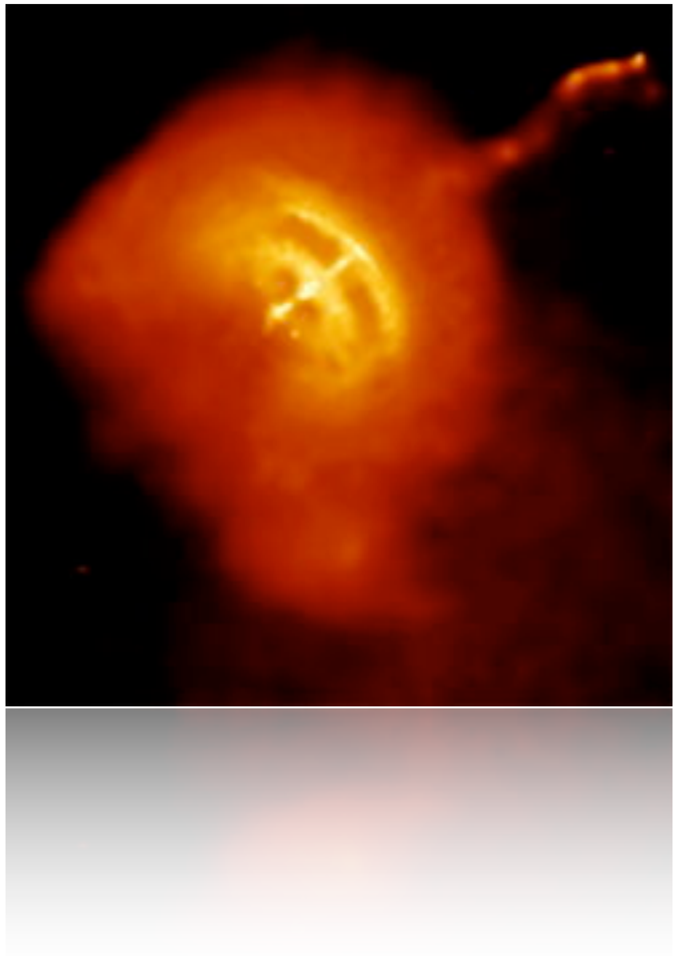


Image: Combined Chandra X-ray Image of Vela Pulsar Jet (Credit: NASA/CXC/PSU/G.Pavlov et al.)

# Summary

- Pulsar glitches: sudden changes in pulsar rotation, could excite fundamental oscillations throughout star
- Oscillations will be damped by gravitational wave emission
  - short, decaying sinusoidal gravitational wave signal at neutron star oscillation frequency
  - Bayesian model selection: logical & intuitive method for identifying a preference for physical models through posterior probability of models.
- Initial application to GW data analysis: choose between signal or noise models
- In case of detection: characterise signal parameters through posterior probability densities
- In case of no detection: set upper limits on (e.g.) amplitude and energy of GWs through integration of posterior probability densities
- Could relate gravitational wave limits to NS oscillation amplitudes
- This has been applied to a search for *f*-mode ring-downs @ 1-3 kHz associated with a glitch in the Vela pulsar during S5

BONUS SLIDES !!!

# Interpreting Upper Limits

- Potential to relate upper limit on gravitational wave energy to physical size of f-mode oscillations:
- Assume **all** of glitch energy goes into exciting l=2, m=0 mode (for simplicity)
- Write down boundary surface of neutron star

$$r(\theta, t) = R_* \left\{ 1 + \frac{a_{20}(t)}{2} \sqrt{\frac{5}{4\pi}} \left[ 3\cos^2(\theta) - 1 \right] \right\}$$

- where

$$a_{20}(t) = \mathcal{A}_{20} \sin(\omega_f t) e^{-t/\tau_f}$$

-Assume constant density*, isotropic emission, compute initial amplitude of GWs:

$$h_0 = 32 \left( \frac{\pi^5}{5} \right)^{1/2} \frac{G}{c^4} \frac{\mathcal{A}_{20} \nu_f^2 R_*^5 \rho_*}{D}$$

- Compute $A_{20}$ for canonical values:

$$A_{20} \sim 10^{-6} \left( \frac{E_{\rm GW}}{10^{-11} M_\odot c^2} \right)^{1/2} \left( \frac{200\,{\rm ms}}{\tau} \right)^{1/2} \left( \frac{2\,{\rm kHz}}{\nu_{20}} \right)^3 \left( \frac{10\,{\rm km}}{R_*} \right)^5 \left( \frac{10^{18}\,{\rm kg\,m^{-3}}}{\rho_*} \right)$$

For 10 km radius, get ~10 cm distortion to neutron star

*Interesting to consider other profiles

# A Bayesian GW Search (2)

## Choice of priors

- Adopt flat priors on signal frequency $f_0$ and decay time $\tau$:

$$p(f_0|\mathrm{M}_1, I) = \frac{1}{f_0^{(\mathrm{upp})} - f_0^{(\mathrm{low})}} \qquad p(\tau|\mathrm{M}_1, I) = \frac{1}{\tau^{(\mathrm{upp})} - \tau^{(\mathrm{low})}}$$

$f_0^{(\mathrm{upp})} = 3$ kHz, $f_0^{(\mathrm{low})} = 1$ kHz $\qquad \tau^{(\mathrm{upp})} = 0.5$ s, $\tau^{(\mathrm{low})} = 0.05$ s

- Use a (normalised) Jeffrey's prior on signal amplitude, $h_0$

$h_0^{(\mathrm{upp})} = 10^{-19}$ Hz$^{-1/2}$, $h_0^{(\mathrm{low})} = 10^{-22}$ Hz$^{-1/2}$

$$p(h_0|\mathrm{M}_1, I) = \frac{1}{\log\left(h_0^{(\mathrm{upp})}/h_0^{(\mathrm{low})}\right) h_0}$$

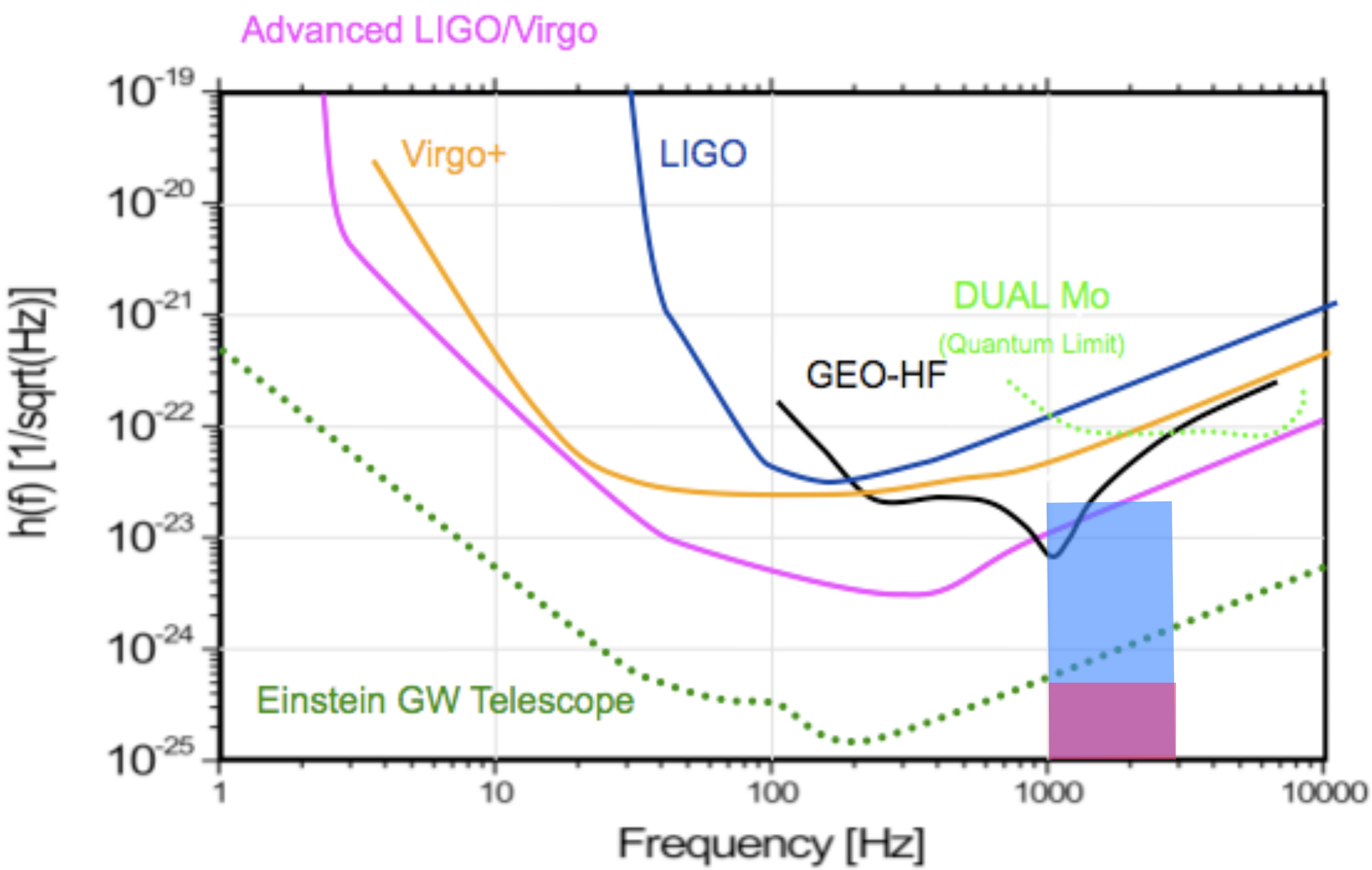
- Pragmatic choice of range to avoid truncating likelihood at high values of $h_0$
- Finite range: ensures correct normalisation
- Uniform in log $h_0$

- Assume no correlation between parameters:

$$p(h_0, f_0, \tau|\mathrm{M}_1, I) = p(h_0|\mathrm{M}_1, I)p(f_0|\mathrm{M}_1, I)p(\tau|\mathrm{M}_1, I)$$

# Pulsar glitches & GWs

- approximate GW amplitude for 100% efficient excitation of fundamental mode oscillations in pulsar @ 293 pc & 15 kpc [i.e., no heating, no other modes excited etc]



# Instrumental Glitch Rejection

- Bayesian model selection can be extended to consider additional hypotheses (Clark et al 2007)

$$O_{123} = \frac{p(M_1|D,I)}{p(M_2|D,I) + p(M_3|D,I)}$$

Where $M_1$ = ring-down
$M_2$ = noise
$M_3$ = sine-Gaussian (say)

- Read this as, *'odds of ring-down model* (M1) *versus noise* (M2) ***or** instrumental glitch* (M3) *model*

- Demonstrate by comparing response to ring-down & sine-Gaussian injections

**Ring-down vs noise** **Ring-down vs noise OR sine-Gaussian**