Reconstruction of core-collapse supernova gravitational wave burst signals based on waveform catalogs

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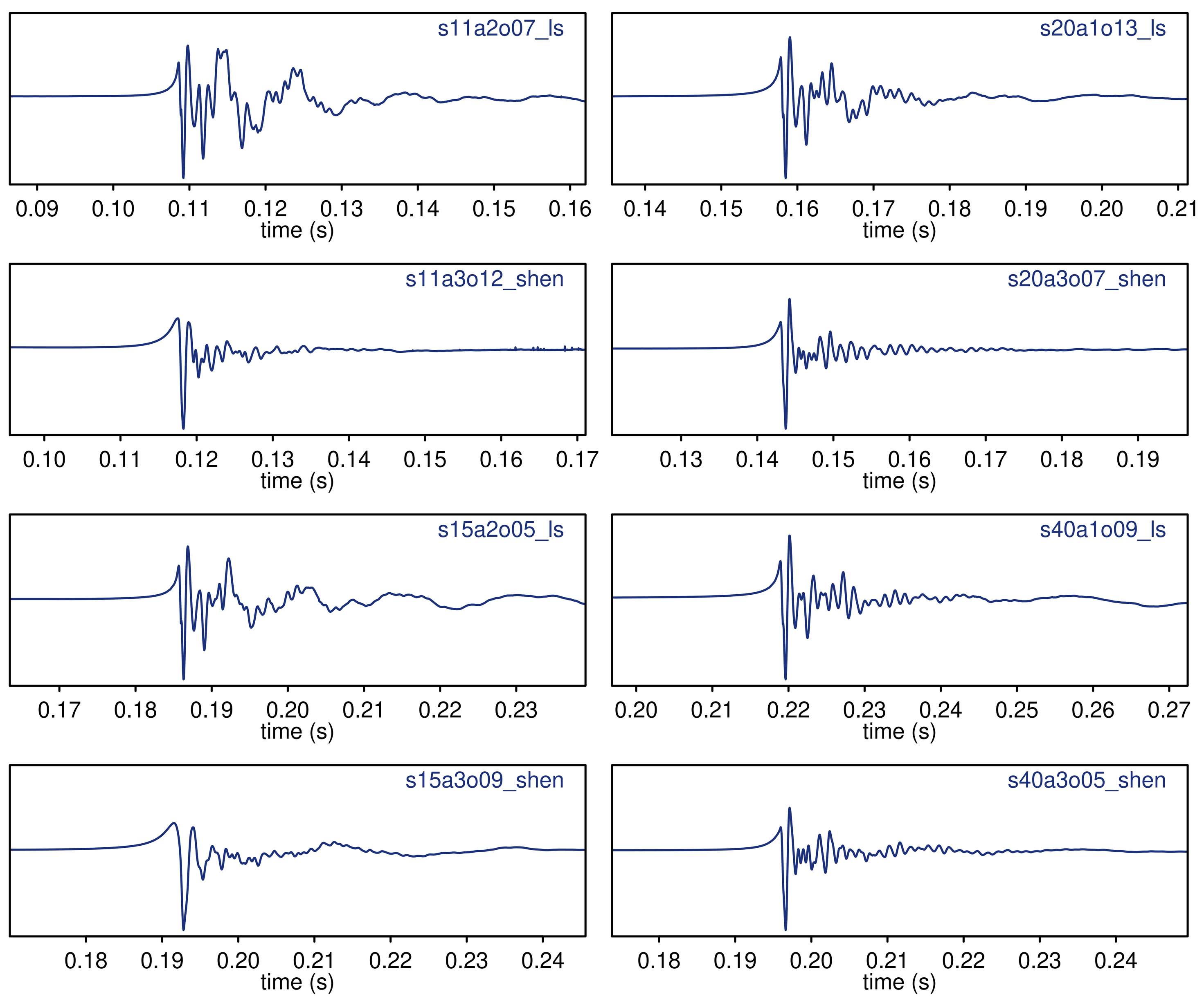
Waveform catalogs from numerical simulations

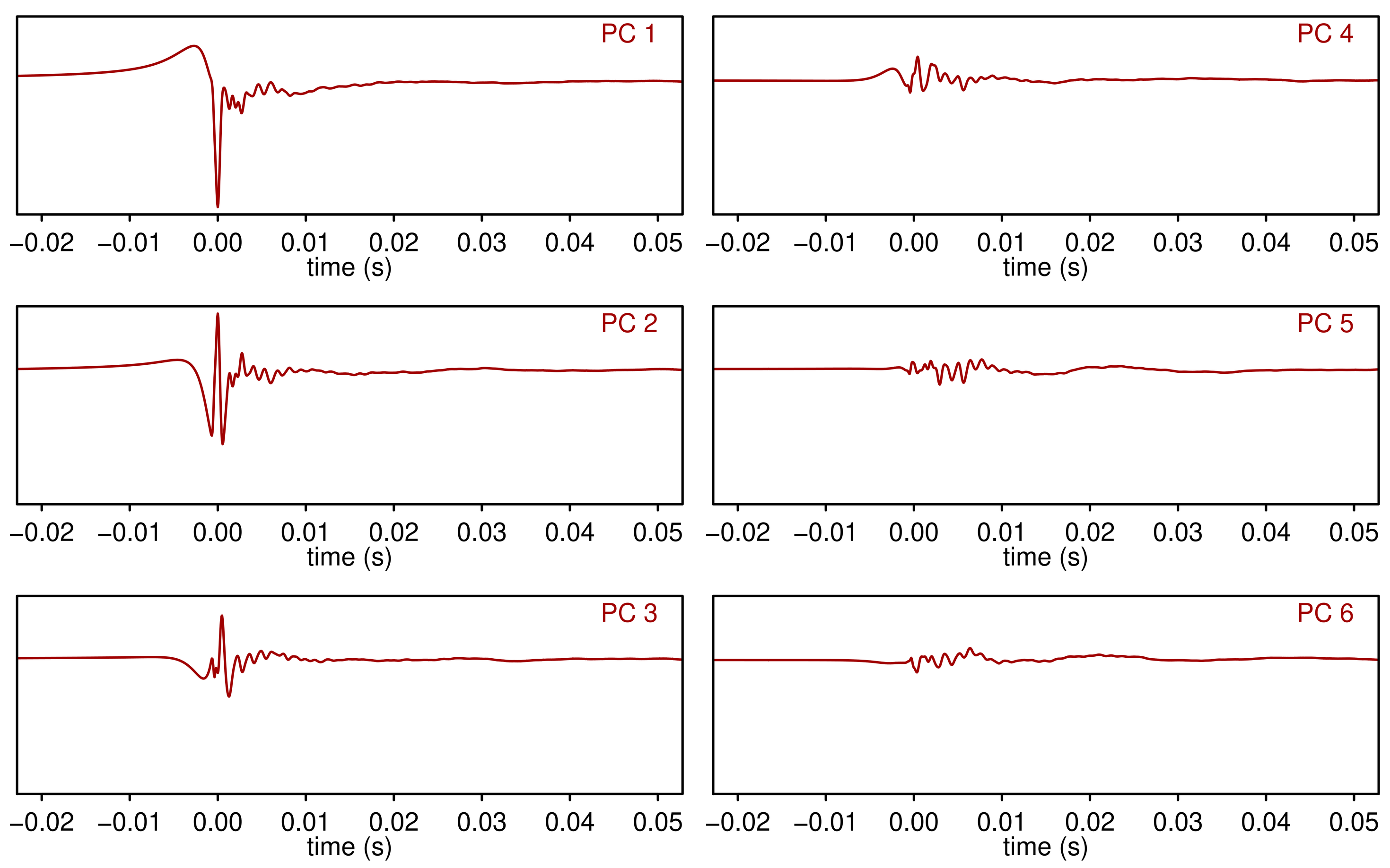
Gravitational wave burst signals from rotating supernova core collapses are derived through computationally expensive numerical simulations:



The above examples of waveforms show that even for a range of input parameters the waveforms exhibit many common features [1, 2].

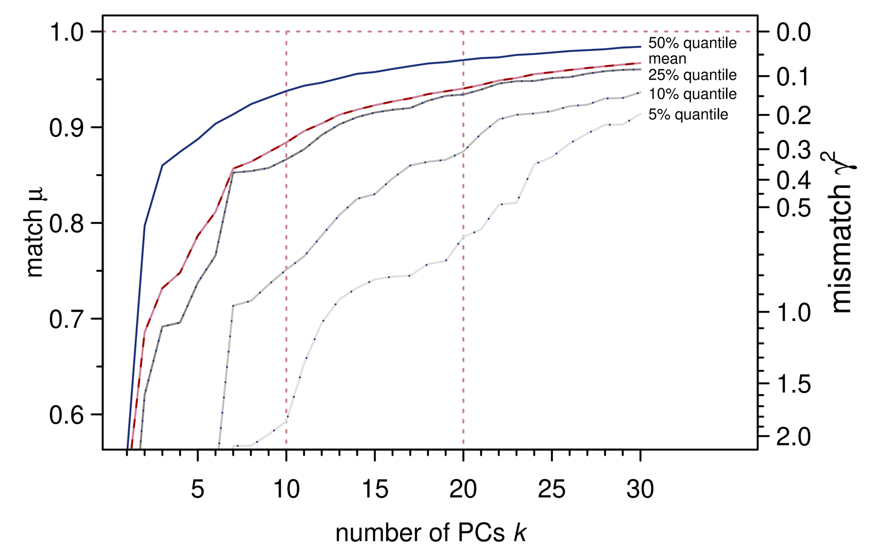
Extracting common features through PCA

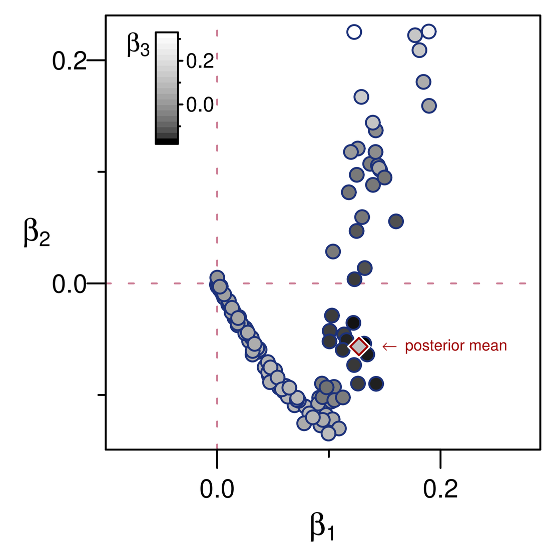
A *principal component analysis* (PCA) allows to extract common features of a large set of waveforms. The plot below shows the first 6 principal components of the above catalogue consisting of 128 waveforms [3].



Waveform reconstruction

Linear combinations of the principal components (PCs) may be used to reconstruct the original waveforms. Few basis vectors are sufficient to yield an accurate match for most signals.





The left plot illustrates the accuracies achieved when reconstructing every single waveform from the original catalog using only a subset of PCs. 10 or 20 (out of 128) basis vectors already yield a great reconstruction accuracy. The blue circles in the right plot show the corresponding first three PC coefficients (β_1 to β_3) for each of the 128 waveforms.

Modelling measured burst signals

The fact that a whole waveform catalog may be reduced to a sparse set of principal components suggests the use of these basis vectors to set up a simplified data analyis model. A measured burst signal is modelled as:

$$\boldsymbol{y} \;=\; \boldsymbol{X}_{(T)}\boldsymbol{\beta} + \boldsymbol{m} + \boldsymbol{\epsilon}$$

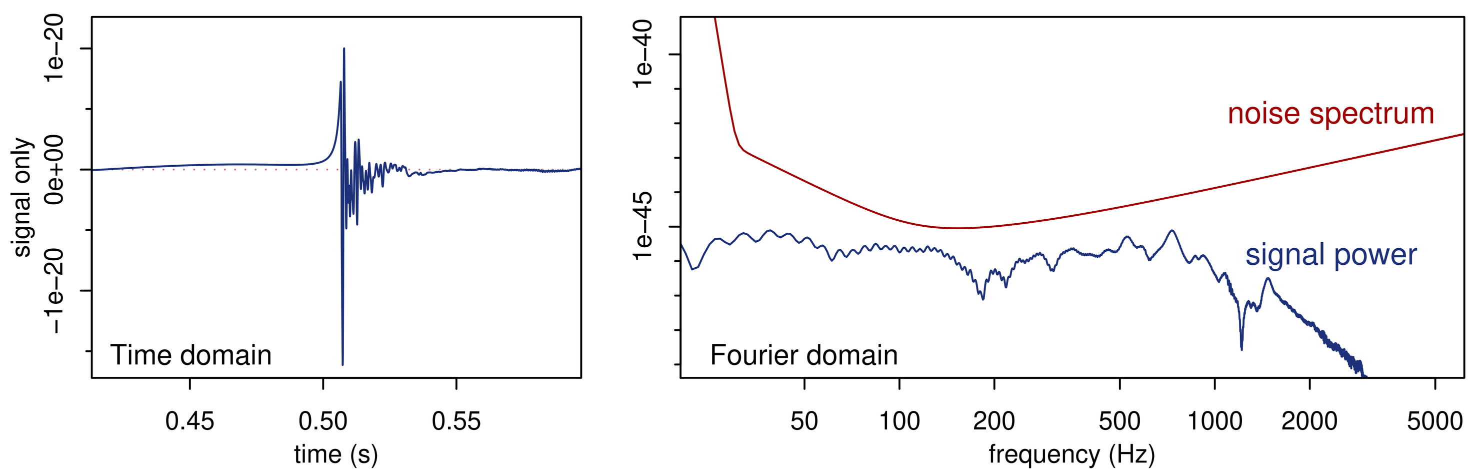
where

- $\boldsymbol{y}$ is the measured data vector
- $\boldsymbol{X}$ is the matrix of basis vectors (PCs)
- $\boldsymbol{\beta}$ is the vector of PC coefficients
- T is the signal arrival time
- $\boldsymbol{m}$ is the mismatch, and
- $\boldsymbol{\epsilon}$ is the measurement noise.

The number of basis vectors k to be used (determining the dimensions of $\boldsymbol{X}$ and $\boldsymbol{\beta}$) is specified in advance. The unknown parameters here are the arrival time T, the coefficients $\boldsymbol{\beta}$, and the mismatch $\boldsymbol{m}$; the noise's power spectral density is given. We assume uninformative, uniform prior distributions for T and $\boldsymbol{\beta}$, and an informative prior ensuring the amount of mismatch to scale with the overall signal amplitude ($\|\boldsymbol{m}\|^2 \propto\!\!\sim \|\boldsymbol{X\beta}\|^2$), as expected from matching PCs with the original catalogue.

Simulated example data

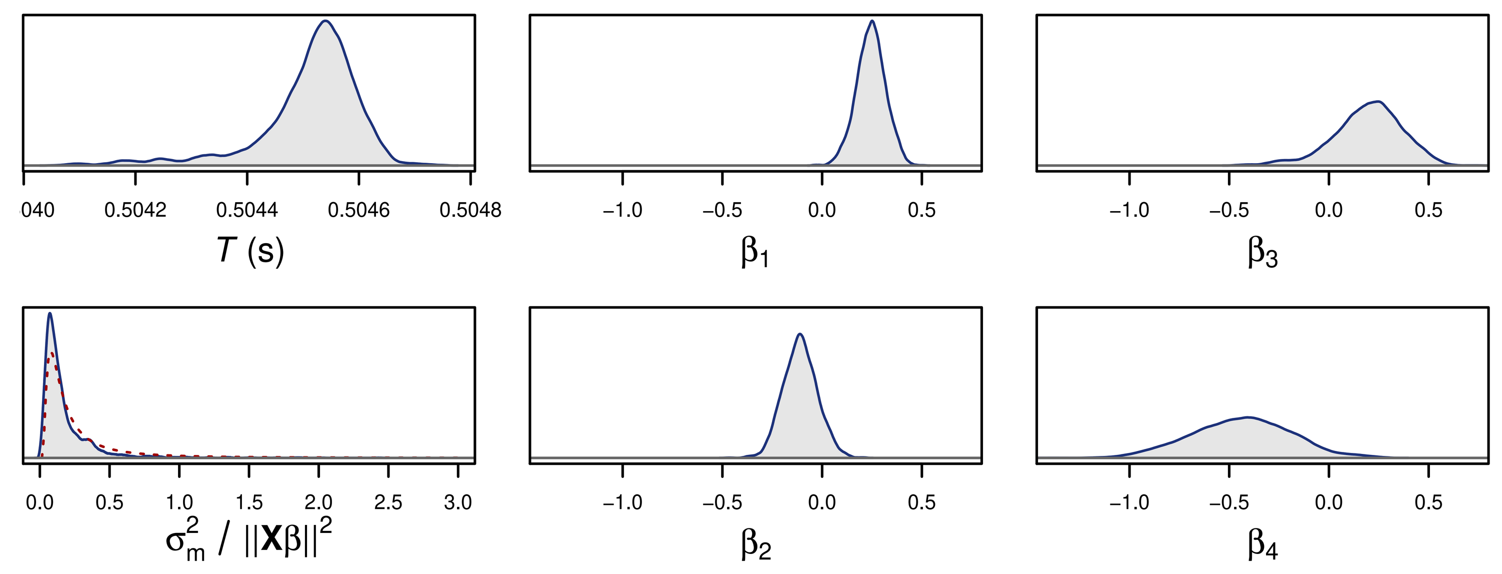
A new signal was generated (using parameter values that did not appear in the original waveform catalogue) and embedded in simulated noise.



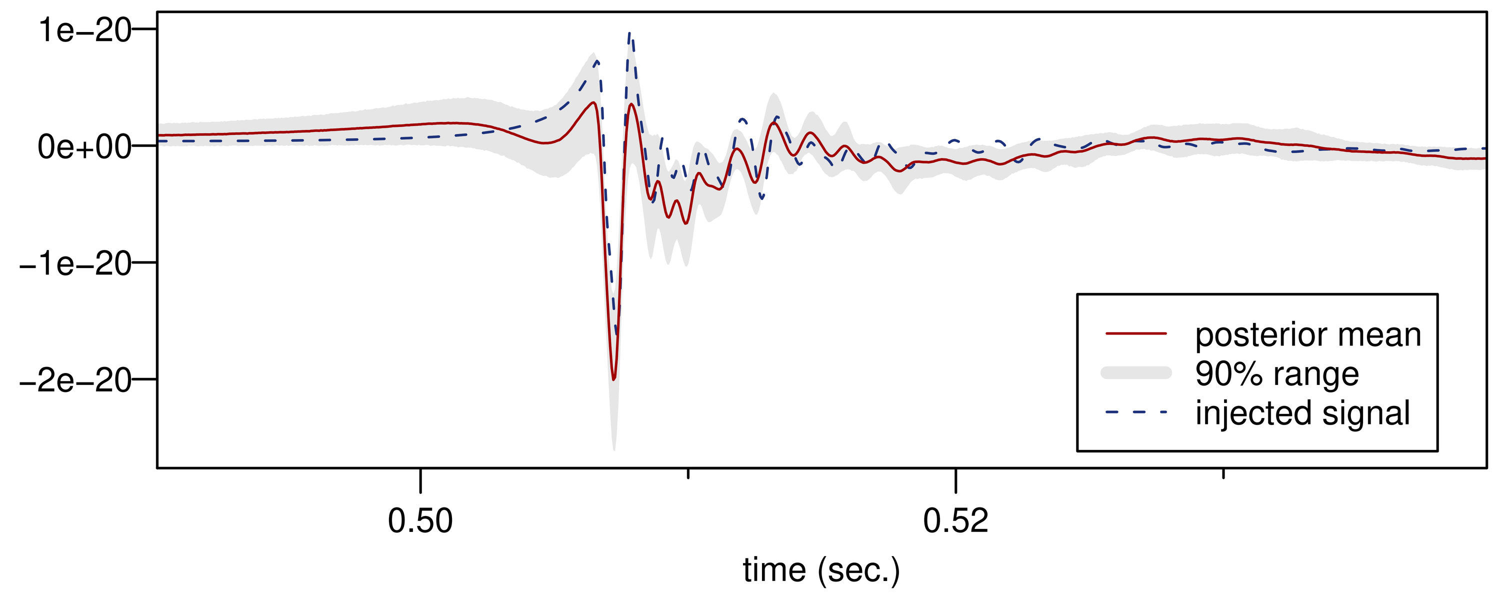
The noise was generated using an *initial LIGO* noise curve, the signal was scaled to an SNR of 10.

Posterior inference

Based on the (simulated) observed data, the posterior distributions of the unknown parameters are derived using a Markov chain Monte Carlo (MCMC) algorithm.



The above plots illustrate the marginal posterior probability densities for some of the parameters. Only the first four coefficients β_i out of a total of $k = 10$ are shown here. The parameters' joint posterior distribution implies the posterior distribution of the waveform at any point in time, and so also allows to reconstruct the actual waveform:



The PC coefficient (β_i) estimates may again be matched against the coefficients from the original catalog; see the posterior mean values shown in the above plot. Ranking by similarity reveals the most similar signals from the catalog:

rank	ID	χ^2	EoS	M_{prog} ($M_\odot$)	$\Omega_{c,i}$ (rad/s)	A (km)	β_{i} / β_{b} / β_{pb} (%)	$\rho_{\max}$ ($\frac{10^{14}}{\mathrm{g\,cm^{-3}}}$)	d_{E} (kpc)
1	`s20a2o13_shen`	10.8	Shen	20	6.45	1000	1.80 / 14.8 / 12.8	2.42	3.50
2	`s11a3o12_shen`	12.8	Shen	11	10.65	500	1.28 / 14.9 / 12.3	2.46	3.46
3	`s11a3o13_shen`	13.7	Shen	11	11.30	500	1.44 / 16.1 / 13.2	2.36	3.37
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	injected signal:		Shen	20	5.48	1000	1.27 / 11.86 / 10.5	2.62	5.17

Outlook

Decomposition of waveform catalogs leads to simple (yet 'physical') models for GW bursts that may be used for analyzing measured signals. We expect that considering larger numbers of waveforms will still increase the number of necessary PCs only slightly. The burst model not only allows to recover waveforms, but also physical properties. This may be useful for parameter estimation or classification of possible event origins.

References

[1] H. Dimmelmeier, C. D. Ott, A. Marek, and H.-T. Janka. Gravitational wave burst signal from core collapse of rotating stars. *Physical Review D*, 78(6):064056, September 2008.

[2] URL `www.mpa-garching.mpg.de/rel_hydro/wave_catalog.shtml`.

[3] I. S. Heng. Rotating stellar core-collapse waveform decomposition: a principal component analysis approach. *Classical and Quantum Gravity*, 26(10):105005, May 2009.

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