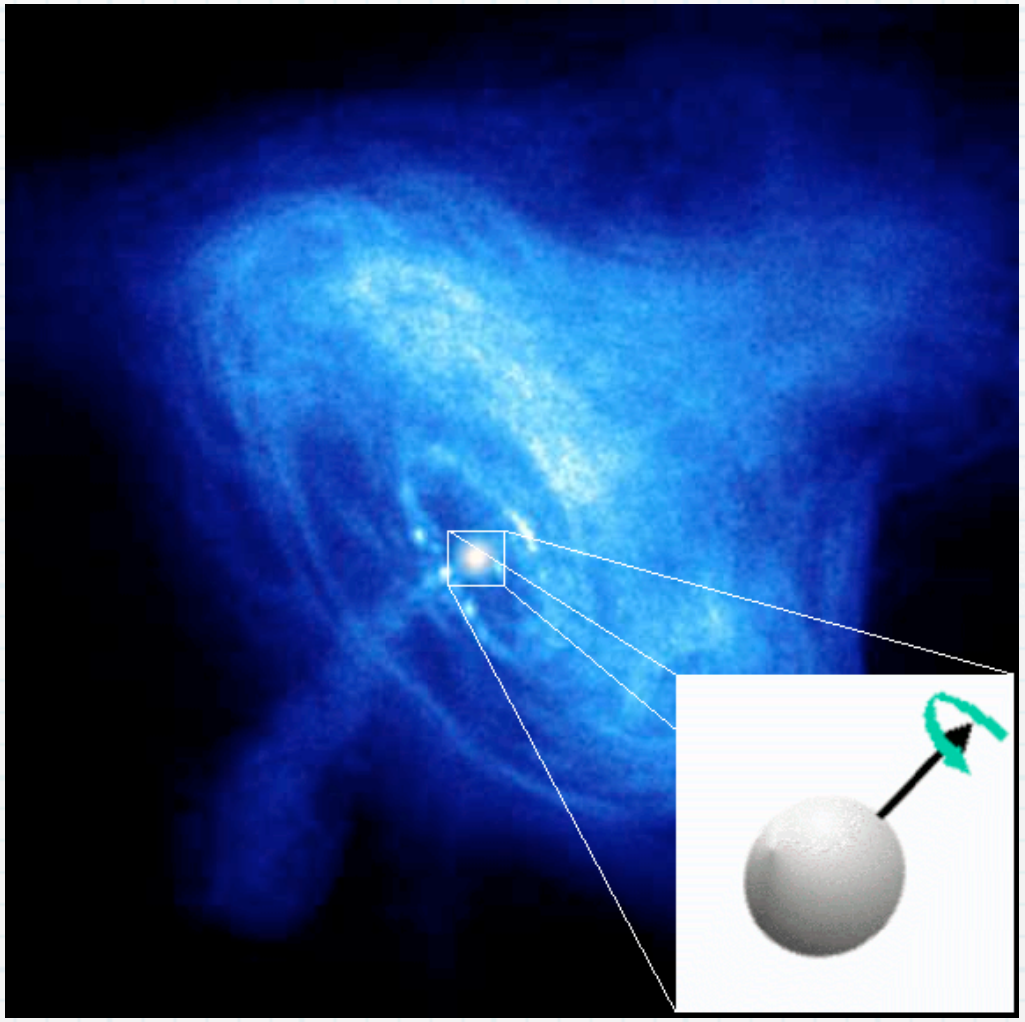
Searching for multi-day transient GWs from NSs



Stefanos Giampanis and Reinhard Prix

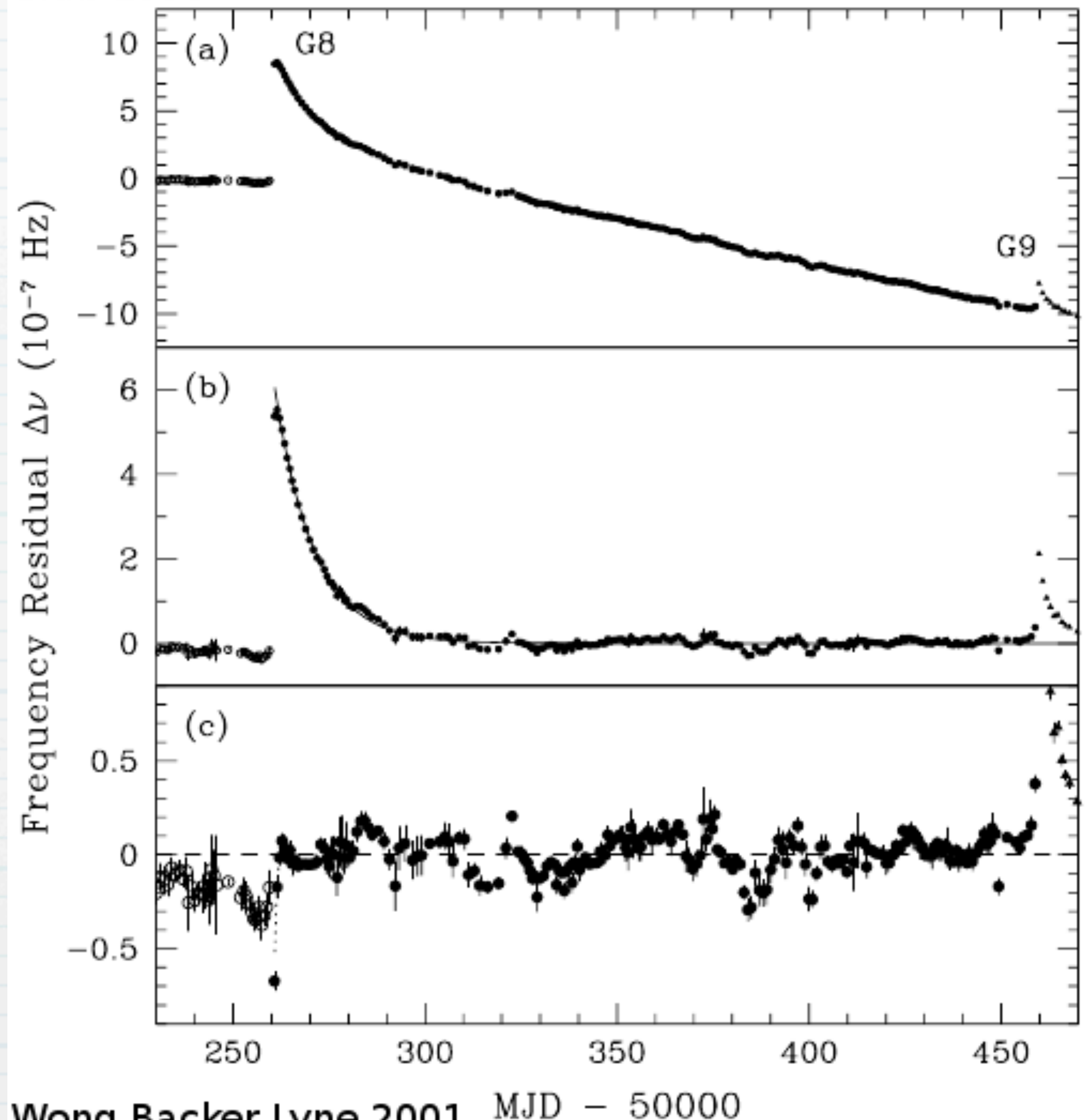
Albert-Einstein-Institute

Hannover

Why "transient" ?

- Previous efforts assumed continuous GWs from NSs
- Transient phenomena are hard to model (predict) but often occur
- Unexplained glitches in NSs rotational rates
- Cover intermediate time scale between "bursts" and continuous GWs (1d - 1 month)

Crab 1996



Wong, Backer, Lyne 2001

Transient GWs model

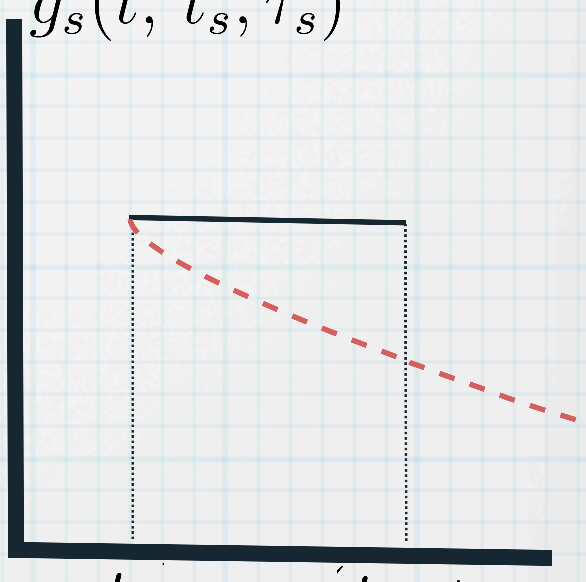
- GW tensor components in NSs rest frame:

$$h_+(\tau) = A_+ \cos\Phi(\tau)\, g_s(\tau), \quad h_\times(\tau) = A_\times \sin\Phi(\tau)\, g_s(\tau)$$

- Phase evolution: $\Phi(\tau) = \phi_0 + \phi(\Delta\tau)$ $\quad \phi(\Delta\tau) = 2\pi \sum_{s=0} \frac{f^{(s)}}{(s+1)!} [\Delta\tau]^{s+1}$

- GW strain:

$$h(t) = \sum_{\mu=1}^{4} g_s(t;\, t_s, \tau_s)\, \mathcal{A}^\mu\, h_\mu(t)$$



- Amplitude/Phase parameters: $\mathcal{A}^\mu = \mathcal{A}^\mu(A_+, A_\times, \psi, \phi_0)$

$$h_1(t) = a(t)\cos\phi(\Delta\tau), \quad h_2(t) = b(t)\cos\phi(\Delta\tau),$$
$$h_3(t) = a(t)\sin\phi(\Delta\tau), \quad h_4(t) = b(t)\sin\phi(\Delta\tau)$$

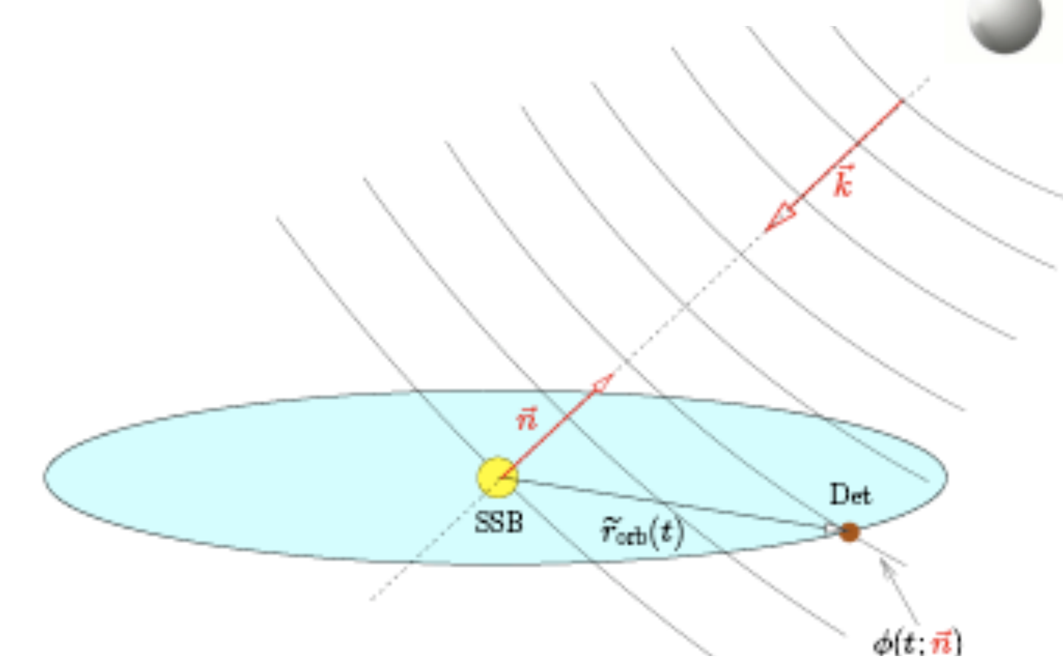
Parameter space

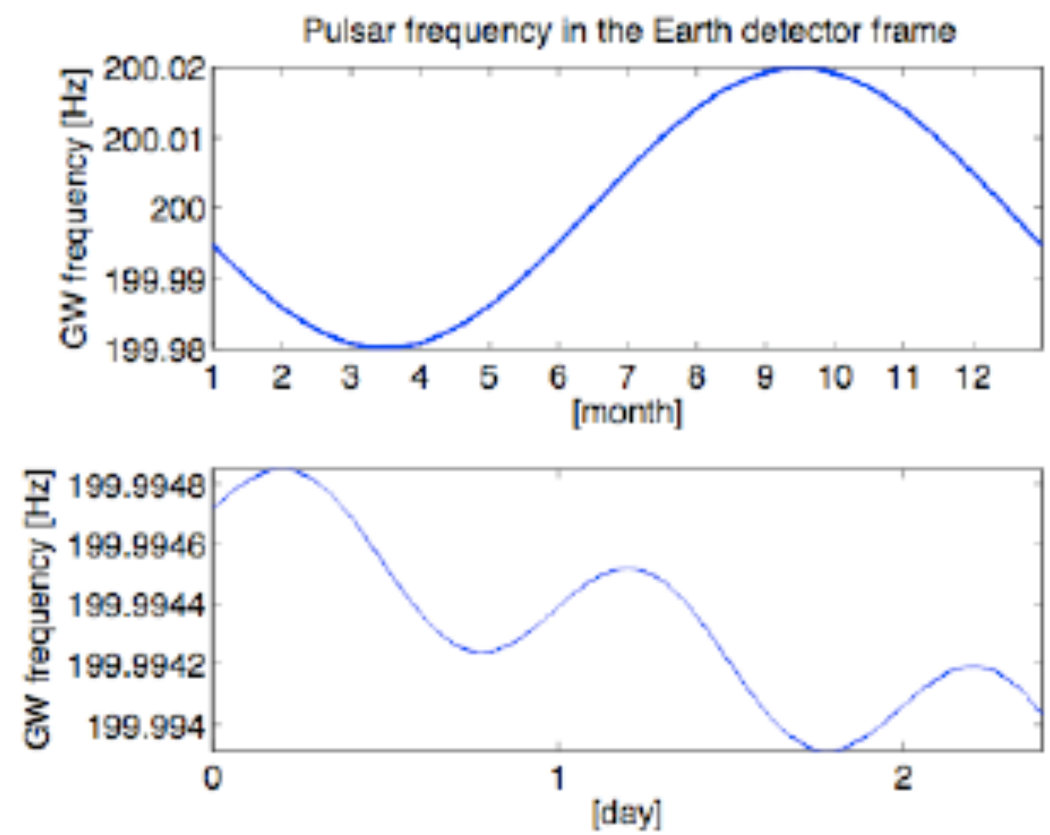
* **Doppler parameters**
  - $\lambda \equiv \{\hat{n}, f^{(s)}\}$ (where $f^{(s)} \equiv d^s f(\tau)/d\tau^s|_{\tau_{\rm ref}}$)

* **Amplitude parameters**
  - $\mathcal{A} = \{A_+, A_\times, \psi, \phi_0\}$

* **“Transient” parameters**
  - $g_0(t; t_0, \Delta T)$

$$\tau(t;\vec{k}) = t - \frac{\vec{r}(t)\cdot\vec{k}}{c} + \text{rel. effects}$$





Matched Filter method

* Correlate a known signal (template) with an unknown signal (data)

* A template is some linear superposition of a vector basis

* Basis vectors: $\boldsymbol{h'}_\mu(t;\lambda,\, t_0, \Delta T) = g_0(\,t_0, \Delta T)\boldsymbol{h}_\mu(t;\lambda)$

* Covariance Matrix : $\boldsymbol{\mathcal{M}}_{\mu\nu} \equiv (\boldsymbol{h'}_\mu|\boldsymbol{h'}_\nu) = (g_0\boldsymbol{h}_\mu|g_0\boldsymbol{h}_\nu)$

* Vectors:

$$\boldsymbol{x}_\mu \equiv (\boldsymbol{x}|\boldsymbol{h'}_\mu) = (\boldsymbol{x}|g_0\boldsymbol{h}_\mu)$$
$$\boldsymbol{s}_\mu \equiv (\boldsymbol{s}|\boldsymbol{h'}_\mu) = (\boldsymbol{s}|g_0\boldsymbol{h}_\mu)$$
$$\boldsymbol{n}_\mu \equiv (\boldsymbol{n}|\boldsymbol{h'}_\mu) = (\boldsymbol{n}|g_0\boldsymbol{h}_\mu)$$

where $\boldsymbol{x}(t) \;=\; \boldsymbol{s}(t) + \boldsymbol{n}(t)$

and $(x|y) \equiv S^{-1}\int_0^\infty x(t)y(t)dt$

Log-Likelihood (F-Statistic)

* Probability of observing the data x(t) given $\mathcal{A}, \lambda, t_0, \Delta T, S$

$$P(\boldsymbol{x}|\mathcal{A}, \lambda, t_0, \Delta T, S) = k\, e^{-\frac{1}{2}(\boldsymbol{x}|\boldsymbol{x})} \, \exp\left[(\boldsymbol{x}|\boldsymbol{s}) - \frac{1}{2}(\boldsymbol{s}|\boldsymbol{s})\right]$$

* Bayes' theorem (and flat priors) gives

$$\log P(\mathcal{A}, \lambda, t_0, \Delta T, |\boldsymbol{x}, S) = \log P_0 + (\boldsymbol{x}|\boldsymbol{s}) - \frac{1}{2}(\boldsymbol{s}|\boldsymbol{s})$$

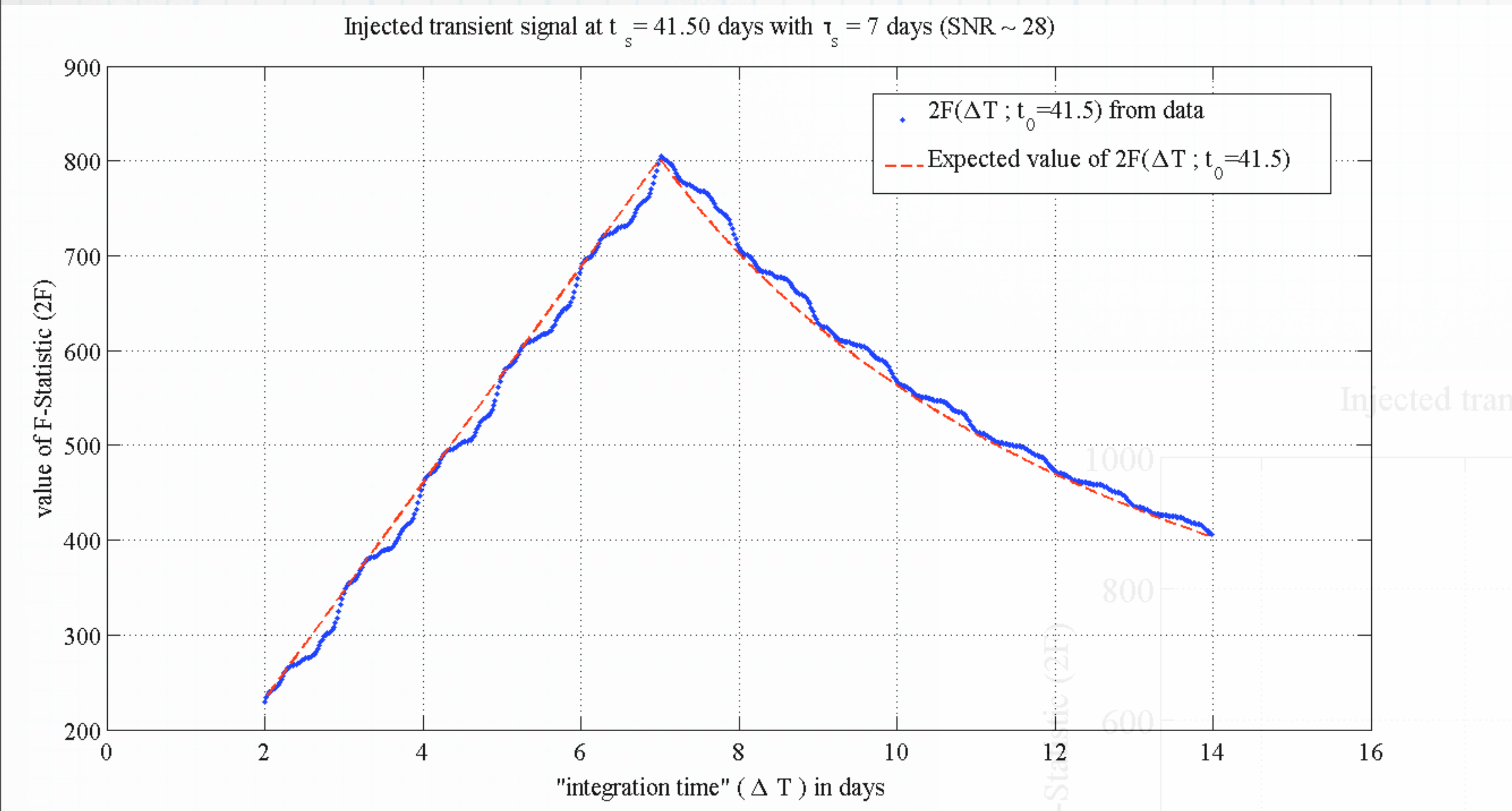
* Marginalize over $\mathcal{A}^\mu$

- $$\{\log P(\lambda, t_0, \Delta T|\boldsymbol{x}, S)\}_{MAX} \quad = \quad \log P_0 \quad + \quad \frac{1}{2}\boldsymbol{x}_\mu \, \boldsymbol{\mathcal{M}}^{\mu\nu} \, \boldsymbol{x}_\nu$$
- "F-Statistic" : $2\mathcal{F}(\lambda, t_0, \Delta T|\boldsymbol{x}) = \boldsymbol{x}_\mu \, \boldsymbol{\mathcal{M}}^{\mu\nu} \, \boldsymbol{x}_\nu$

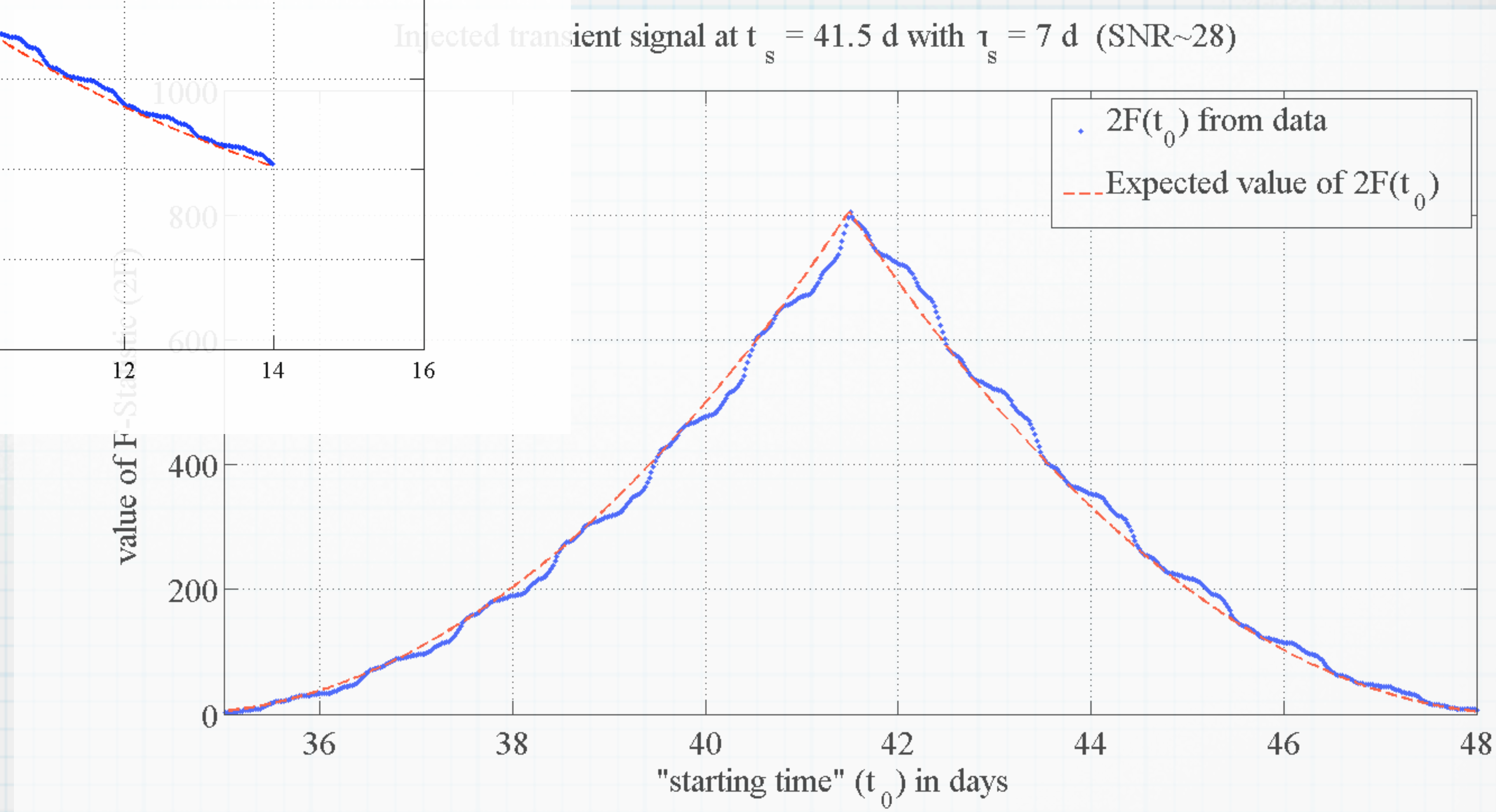
Expected value of F-Statistic

- $E[2\mathcal{F}] \quad = \quad 4 + s_\mu \, \mathcal{M}^{\mu\nu} \, s_\nu \xrightarrow{\text{rect window}} 4 + (\frac{\tau_1 - \tau_0}{\Delta T})^2 \, \frac{\Delta T}{S_h} [\mathcal{A}^\mu \langle \boldsymbol{h}_\mu \boldsymbol{h}_\nu \rangle \mathcal{A}^\nu]$

where $\tau_0 = max(t_0, t_s)$ **and** $\tau_1 = min(t_s + \tau_s, t_0 + \Delta T)$



E[2F] (ΔT=const.)



E[2F] (t_0=const.)

Statistics - Hypothesis testing

- 2 hypotheses:

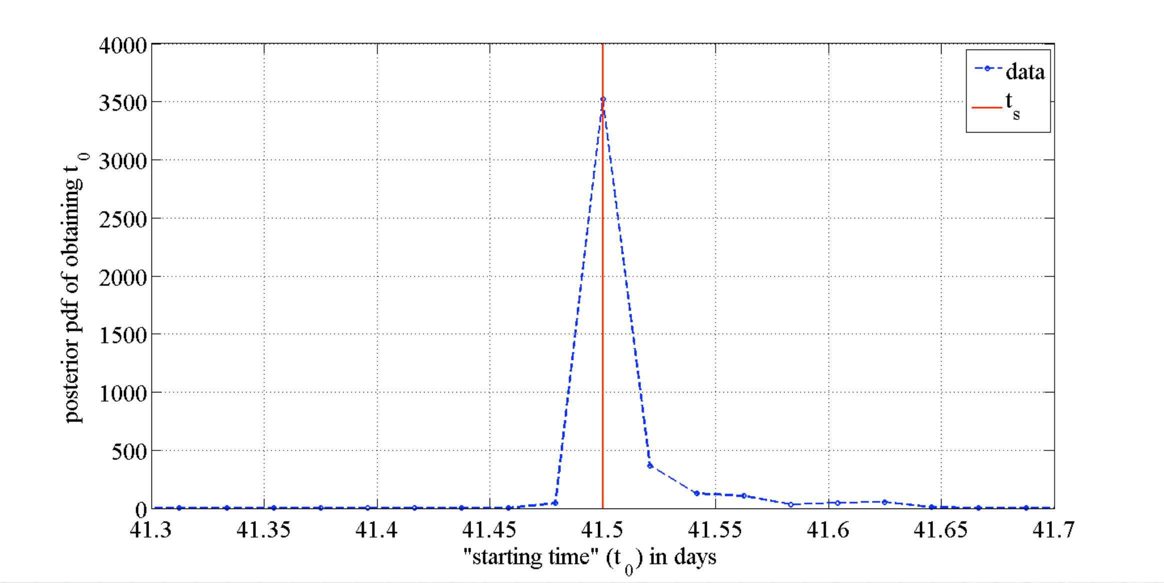
  - null hypothesis
  - signal case

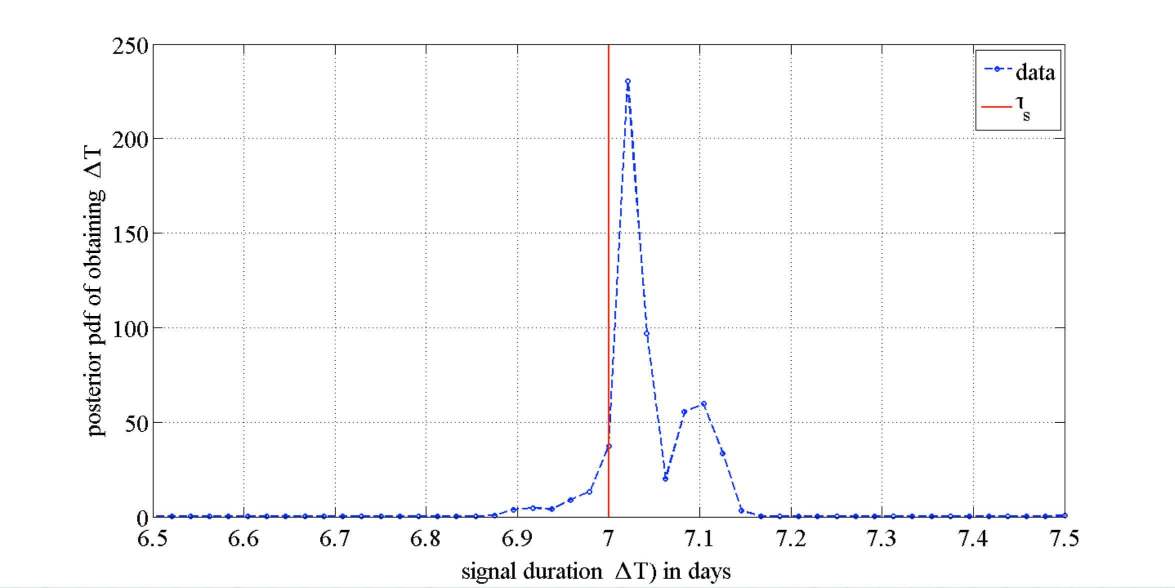
$$\mathcal{H}_0 : \boldsymbol{x}(t) = \boldsymbol{n}(t)$$
$$\mathcal{H}_1 : \boldsymbol{x}(t) = \boldsymbol{n}(t) + \boldsymbol{s}(t; \mathcal{A}, \lambda, t_0, \Delta T)$$

- Testing:
  - Odds ratio
  - Bayes factor
  - Posterior PDFs

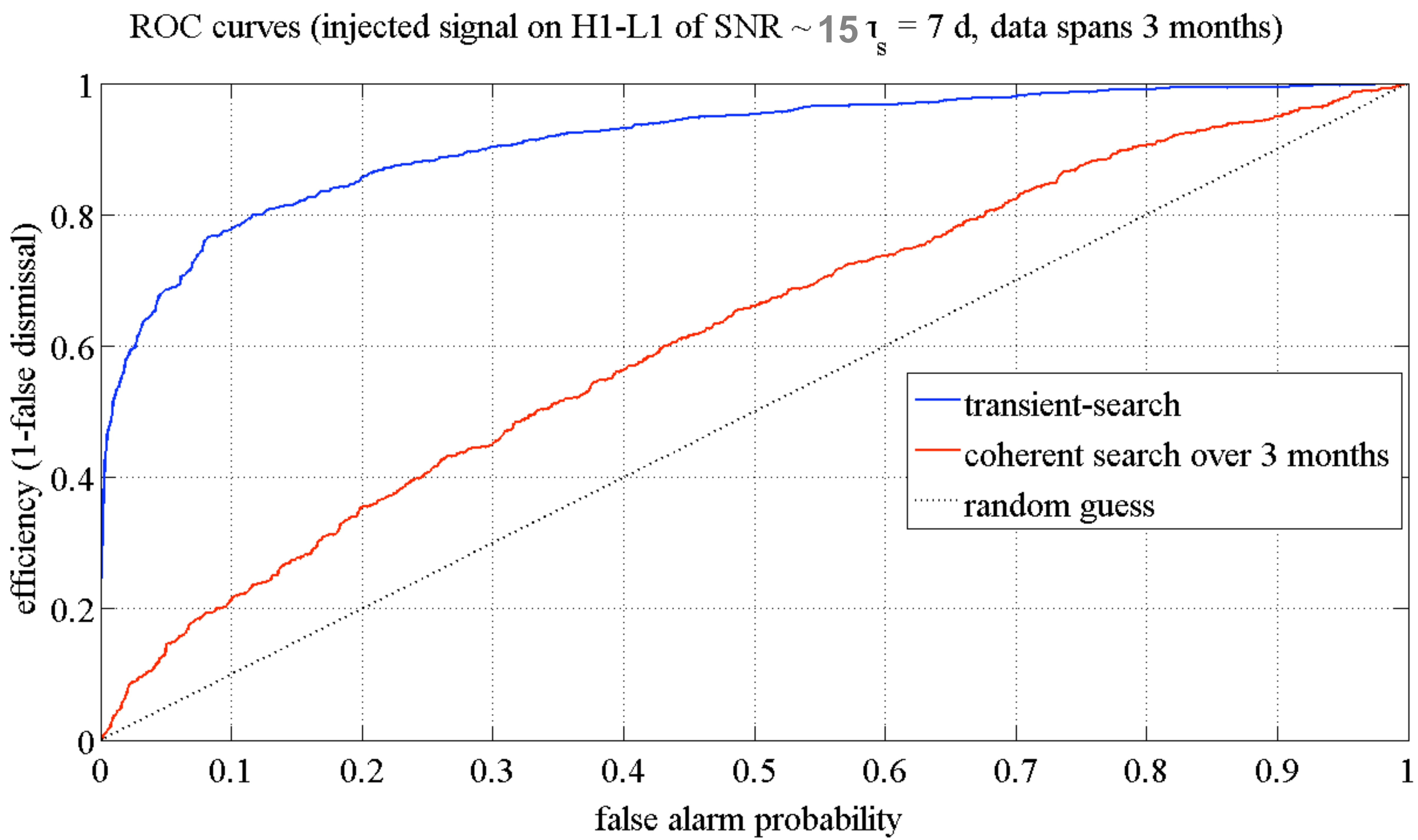
$$O_{10}(x|I) = \frac{P(\mathcal{H}_1|xI)}{P(\mathcal{H}_0|xI)} = \frac{pdf(x|\mathcal{H}_1 I)}{pdf(x|\mathcal{H}_0 I)} \frac{P(\mathcal{H}_1|I)}{P(\mathcal{H}_0|I)}$$

$$\mathcal{B}_{10} \propto \int e^{\mathcal{F}(x,\lambda,t_0,\Delta T)} P(t_0, \Delta T|\mathcal{H}_1) dt_0 \, d\Delta T$$





“transient” search vs. “coherent” search



Summary

* Developed a Bayesian (Odds-ratio) search method for a "transient" (1d-1month) GW signal from known pulsars

* Classical (frequentist) F-Statistic can be derived in a Bayesian framework using flat priors on $\mathcal{A}^{\mu}$s (R. Prix to appear in CQG)

* Method is multi-IFO compatible

* Can be extended in "directed" searches (known sky position for a pulsar but searching for a signal in a frequency band)