Gravitational waves from pulsar glitches

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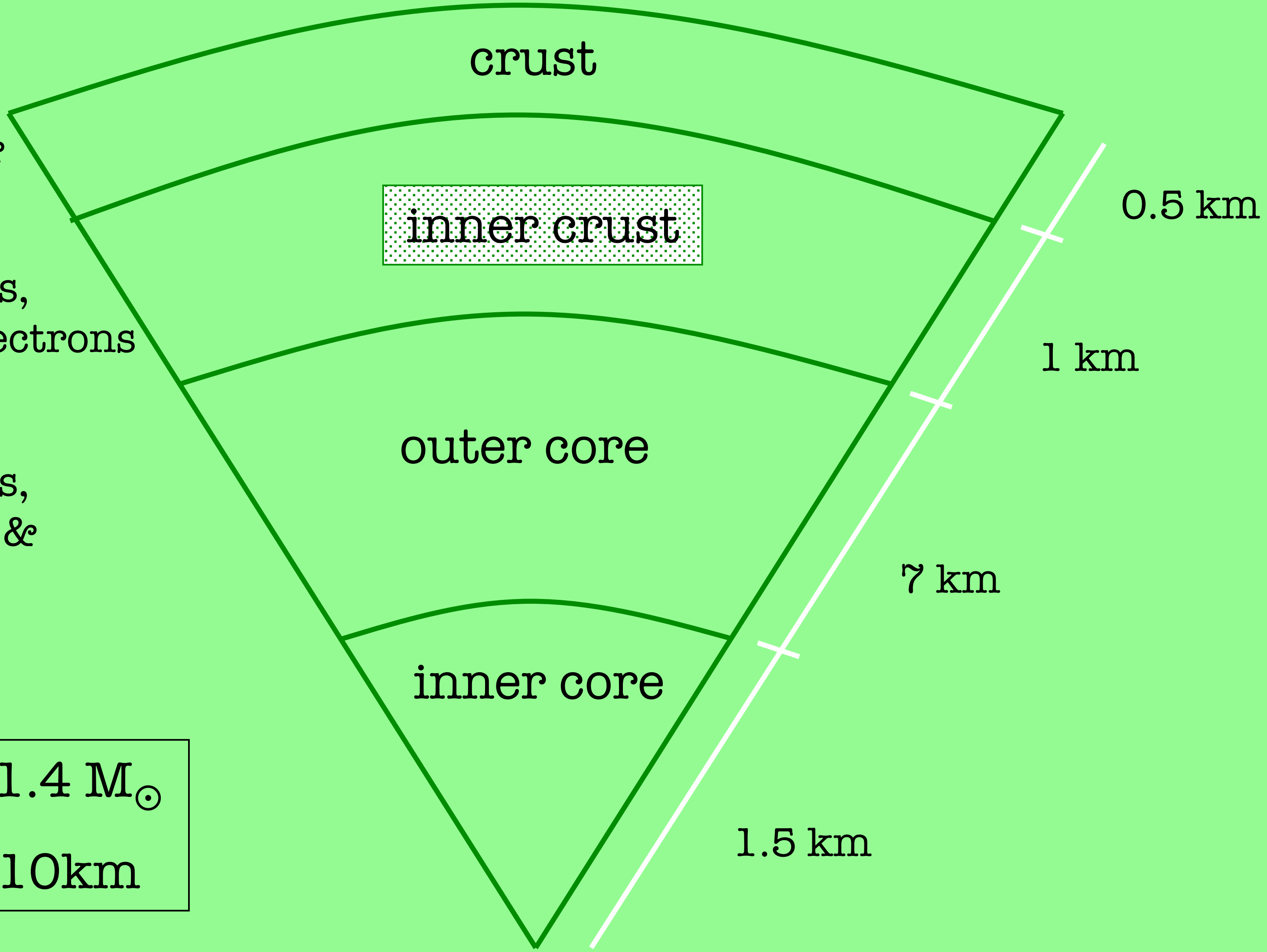
In the next 50 minutes…

- Neutron star basics
- Pulsar glitches, glitch statistics (& GWs)
- Superfluids and vortices (& GWs)
- Glitch models:
  - Avalanches
  - Coherent noise
  - Quantum mechanical (GPE) model
- Gravitational waves from glitches

Neutron star composition



crust

electrons &
ions

inner crust

SF neutrons,
nuclei & electrons

outer core

SF neutrons,
SC protons &
electrons

inner core

0.5 km

1 km

7 km

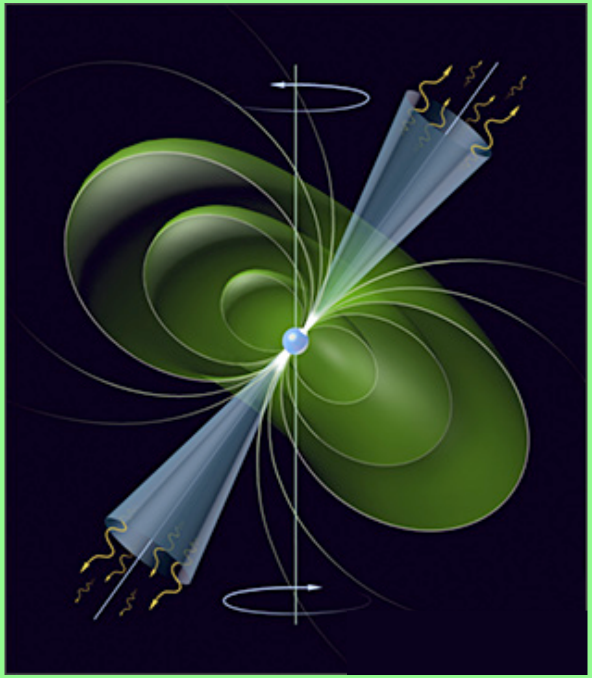
1.5 km

Mass = $1.4\ M_\odot$

Radius = 10km

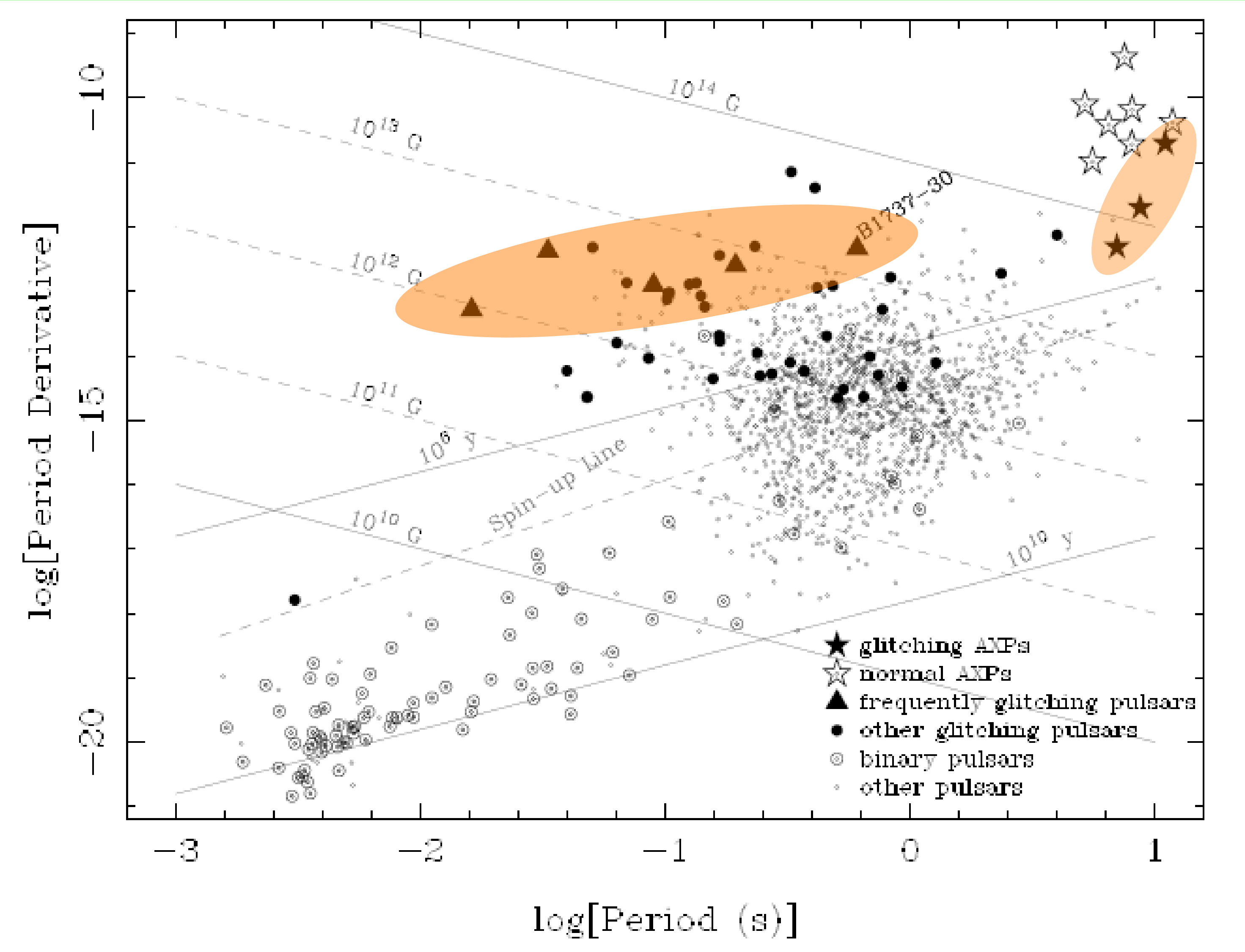
What we know

- Pulsars are neutron stars that emit beams of radiation from magnetic poles.
- Pulsars are extremely reliable clocks (ΔTOA≈100ns).
- Glitches are sporadic changes in ν (↑), and dν/dt (↑ or ↓).
- Some pulsars glitch quasi-periodically, others glitch intermittently.
- Of the approx. 1500 known pulsars, 9 have glitched at least 5 times..
  - Some evidence for age-dependent glitch activity.



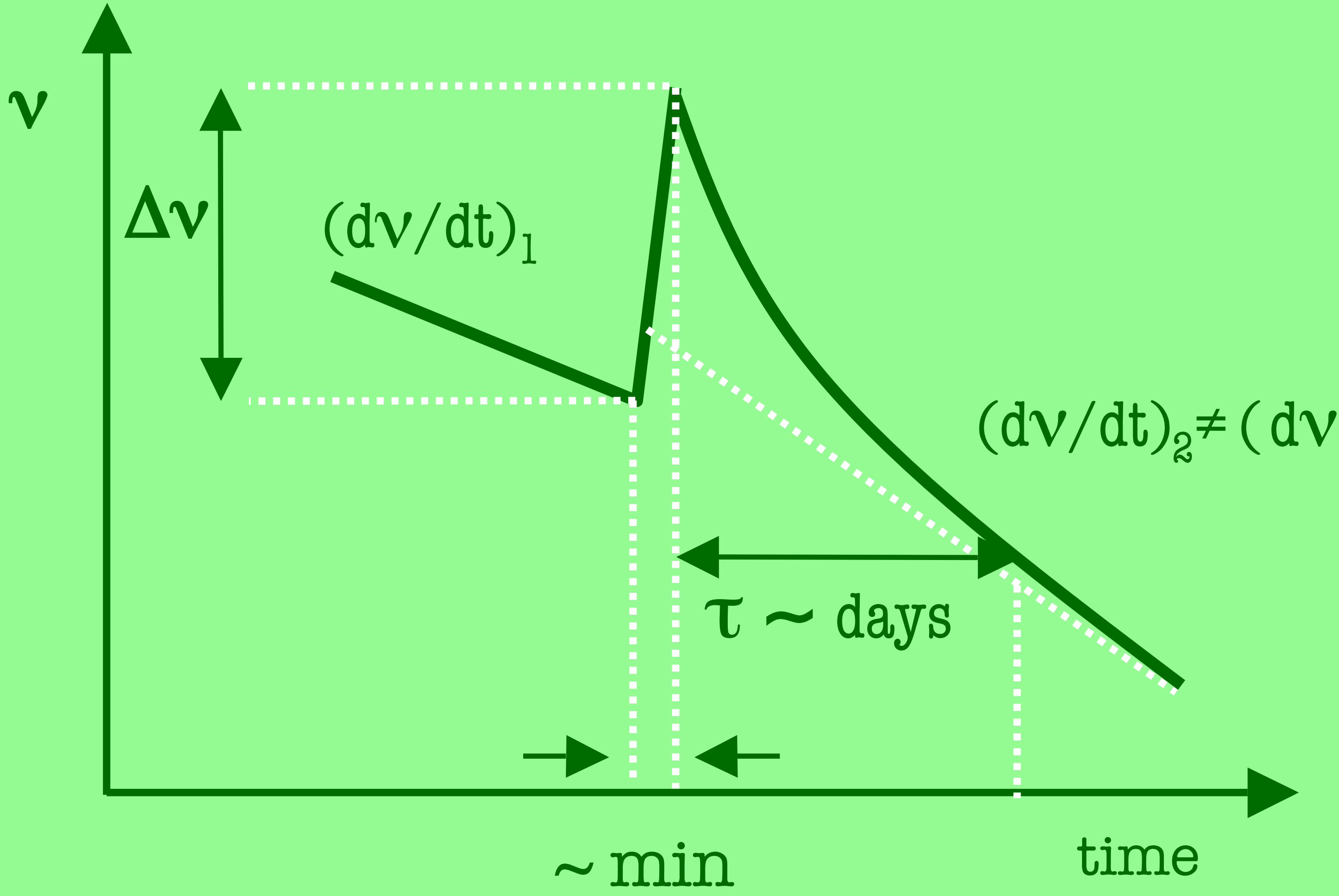
$$\begin{aligned} 0.08\,\mathrm{Hz} &< \nu < 700\,\mathrm{Hz} \\ -3.8\times 10^{-10}\,\mathrm{Hz\,s^{-1}} &< \dot{\nu} < -1\times 10^{-18}\,\mathrm{Hz\,s^{-1}} \end{aligned}$$

Glitching pulsars



Zou *et al.*, MNRAS 2008

Anatomy of a glitch



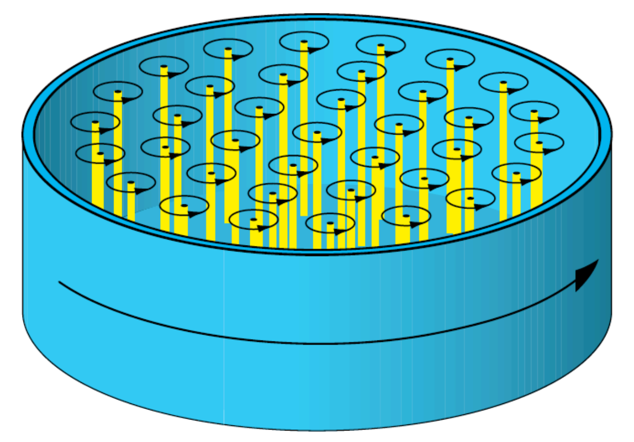
A superfluid interior?

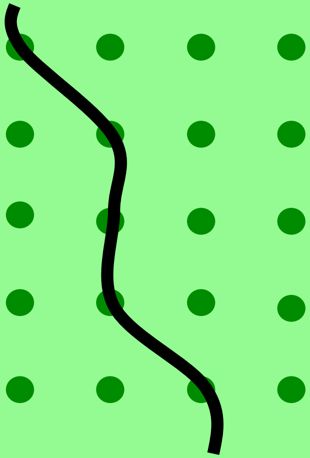
- Post-glitch relaxation slower than for normal fluid:
  - Coupling between interior and crust is weak.
- Nuclear density, temperature below Fermi temperature.
- Spin-up during glitch is *very* fast (<100 s).
  - NOT electomagnetic torque

→Interior fluid is an inviscid (frictionless) **superfluid**.

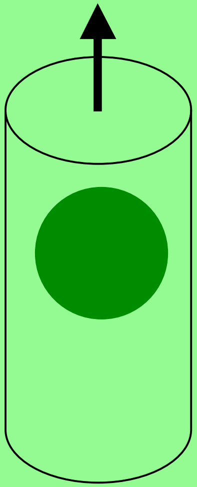
Superfluids & vortices

- SF doesn't 'feel' slow rotation of container
- Above ν_{crit} SF rotates via vortices
  - quantum of circulation
  - 1/r velocity field per vortex
- Vortices form **Abrikosov** lattice
- ν_{SF} determined by vortex density
- <L> determined by vortex positions





- Vortex core is empty
- Superposition of vortex & nucleus minimizes volume from which SF is excluded
- Pinning is the minimum energy state



GWs three ways

- Strongest signal from time-varying current quadrupole moment (s)

$$h \propto \frac{\partial^2}{\partial t^2} \int dV \, \mathrm{vorticity}$$

- **Burst** signal (this talk):
  - Vortex rearrangement → changing velocity field
- Post-glitch **ringing**:
  - Viscous component of interior fluid adjusts to spin-up
- **Stochastic** signal:
  - Turbulence (eddies) [Melatos & Peralta (2009)]

Pulsar glitch statistics

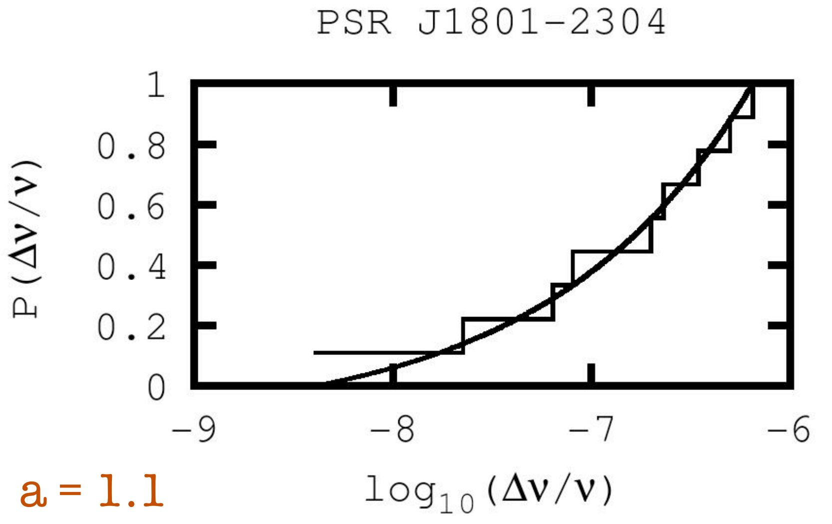
Melatos, Peralta & Wyithe, **672**, ApJ (2008)

- Glitch sizes vary up to 4 decs in $\Delta\nu/\nu$
- Fractional glitch size follows a *different* power law for each pulsar.

$$p(\Delta\nu/\nu) \propto (\Delta\nu/\nu)^{-a}$$

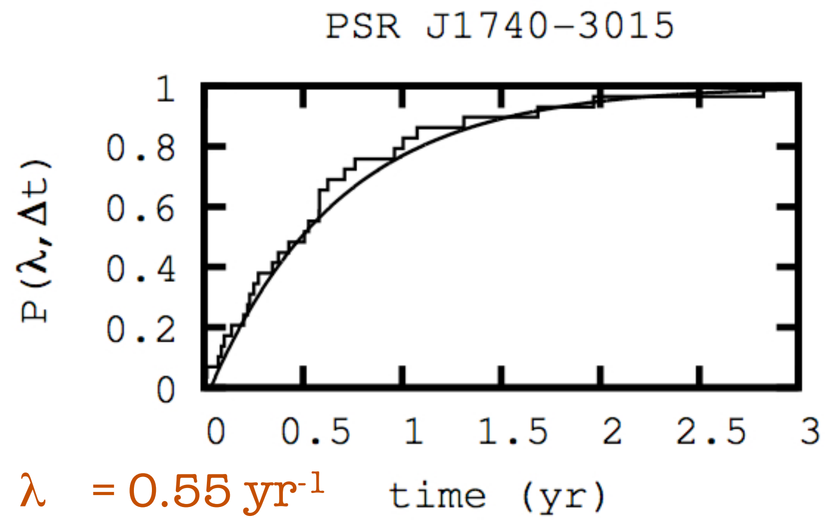
- Waiting times between glitches obey Poissonian statistics.

$$p(\lambda, \Delta t) = \lambda \exp(-\lambda \Delta t)$$



$a = 1.1$

Cumulative fractional glitch size

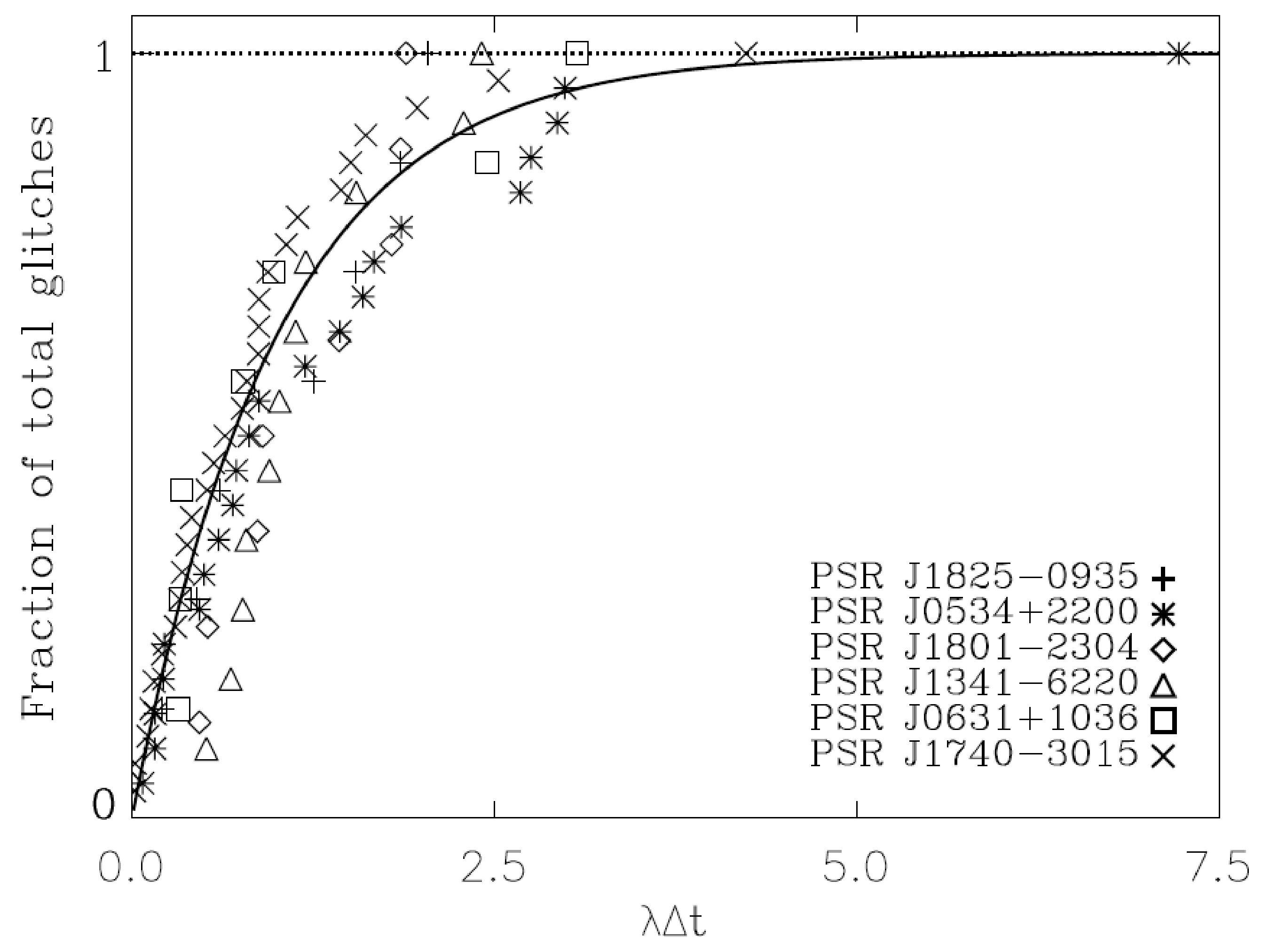


$\lambda = 0.55\ \mathrm{yr}^{-1}$

Cumulative waiting time

Poisson waiting times

Warszawski & Melatos, MNRAS (2008)



The unpinning paradigm

Anderson & Itoh, 1975, *Nature*, 256, 25

1. Nuclear lattice + neutron superfluid (SF).
2. Rotation of crust → vortices form → SF rotates.
3. Pinned vortices co-rotate with crust.
4. Differential rotation between crust and SF → Magnus force.
5. Vortices unpin → transfer of L to crust → crust spins up.

Some flaws…

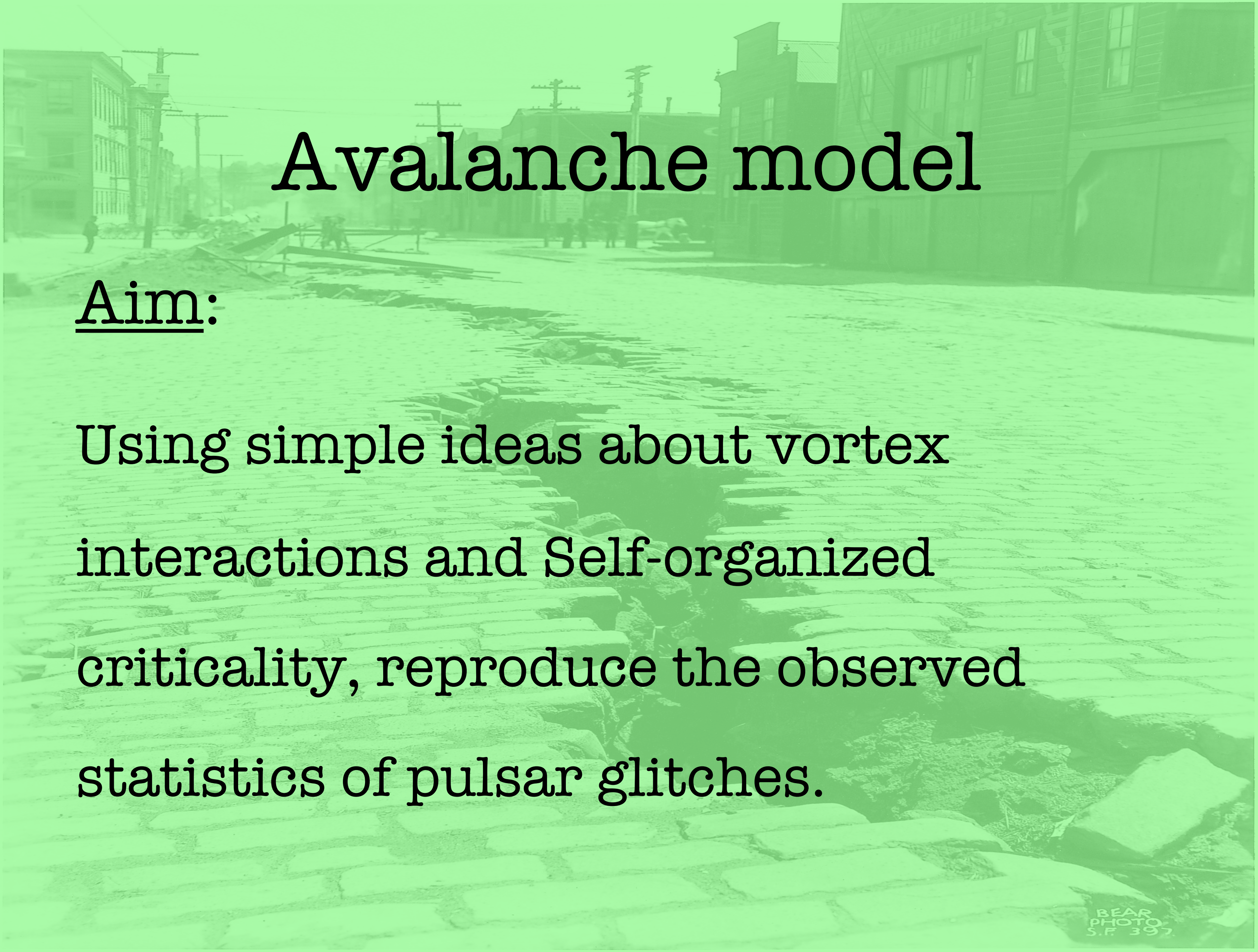
- To what do the vortices pin?
- Vortex separation ≈ 1cm (>> pinning site spacing)
  - Any nuclear lattice site → near continuous dist’n
  - Faults in the crust → inhomogeneous dist’n
- Why doesn’t this result in periodic glitches?
  - If pinning strength is same everywhere and stress builds up uniformly…

→ glitches should all be same size.

Ignores important **collective** dynamics - challenge!

Reality check

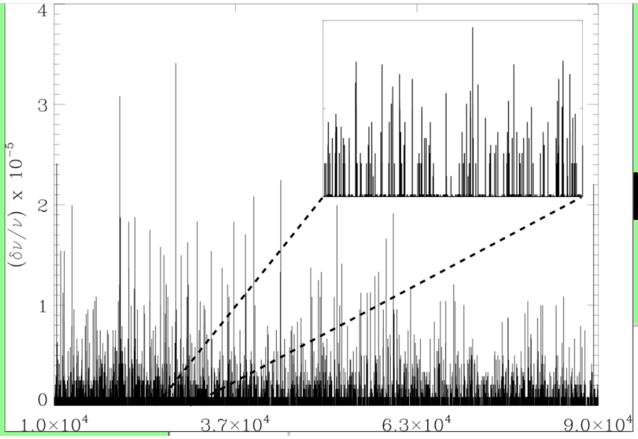
- Superfluid flow should be turbulent:
  - Vortices form a *tangle* rather than a regular array.
- Simulations show that meridional flows develop
  - 3D is important here! (Peralta *et al.* 2005, 2006)
- How does superfluid spindown get communicated to crust?
  - Back-reaction on pinning lattice?
- Role of proton vortices, magnetic fields…



Avalanche model

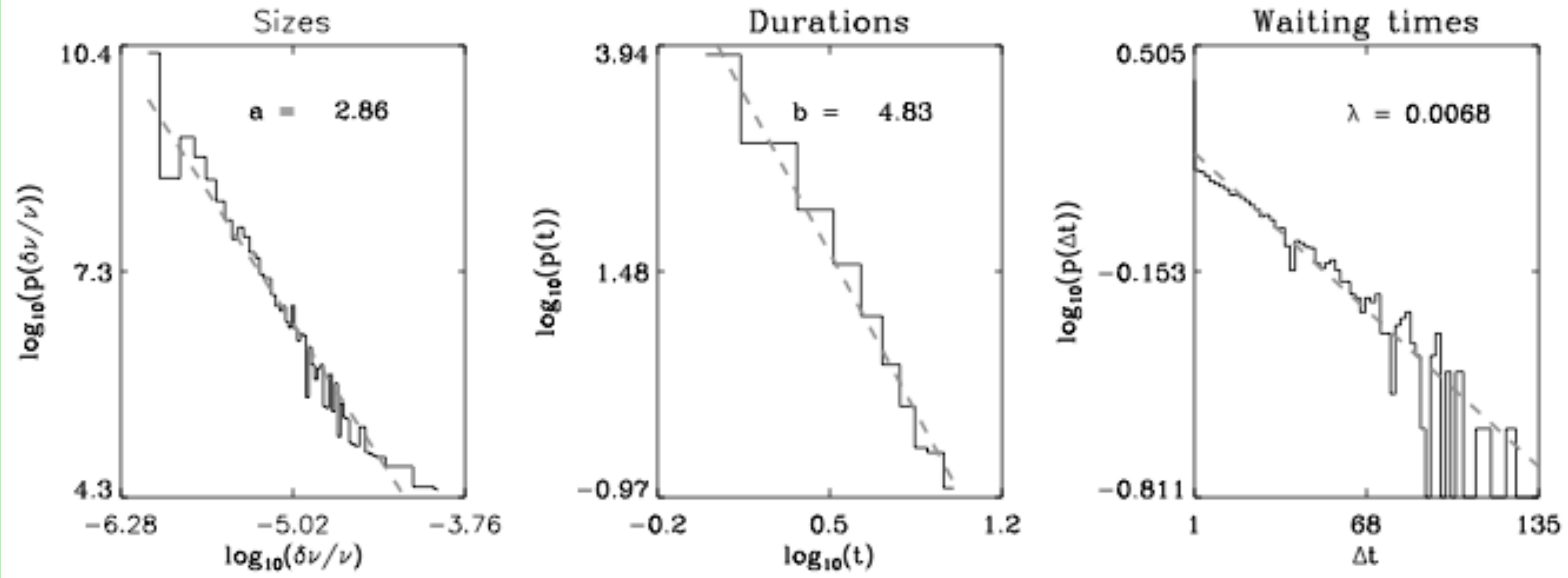
Aim:

Using simple ideas about vortex interactions and Self-organized criticality, reproduce the observed statistics of pulsar glitches.



mulated avalanches

Warszawski & Melatos, MNRAS (2008)



- Power laws in the glitch size and duration support scale invariance.
- Poissonian waiting times supports statistical independence of glitches.

time

Coherent noise

Melatos & Warszawski, ApJ (2009)

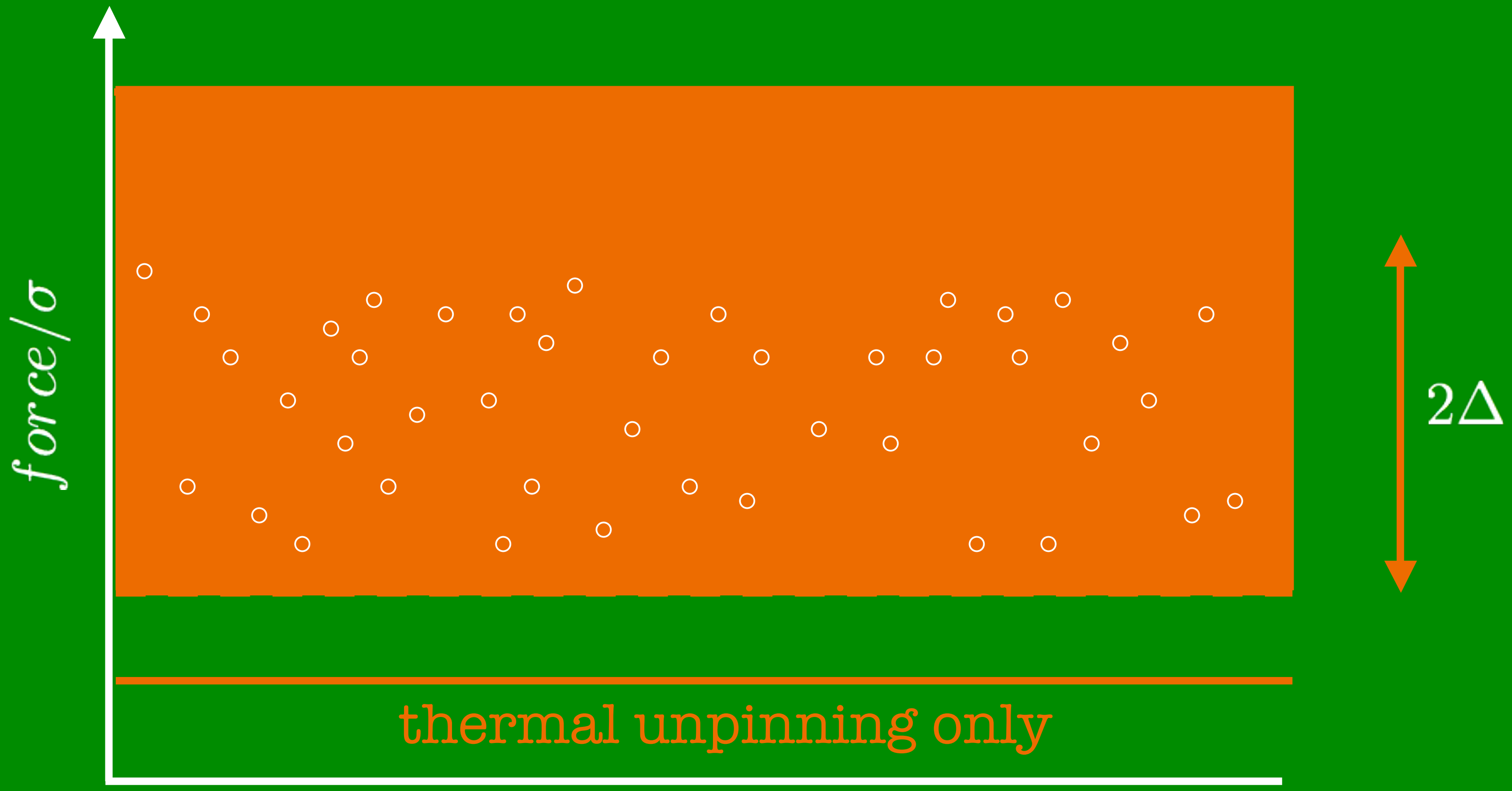
Sneppen & Newman PRE (1996)

- Scale-invariant behaviour without macroscopically inhomogeneous pinning distribution .
- Pinning strength varies from site to site, drawn from top-hat distribution centred on F_0.
- Uniform Magnus force drawn from probability distribution based only on spin-down:

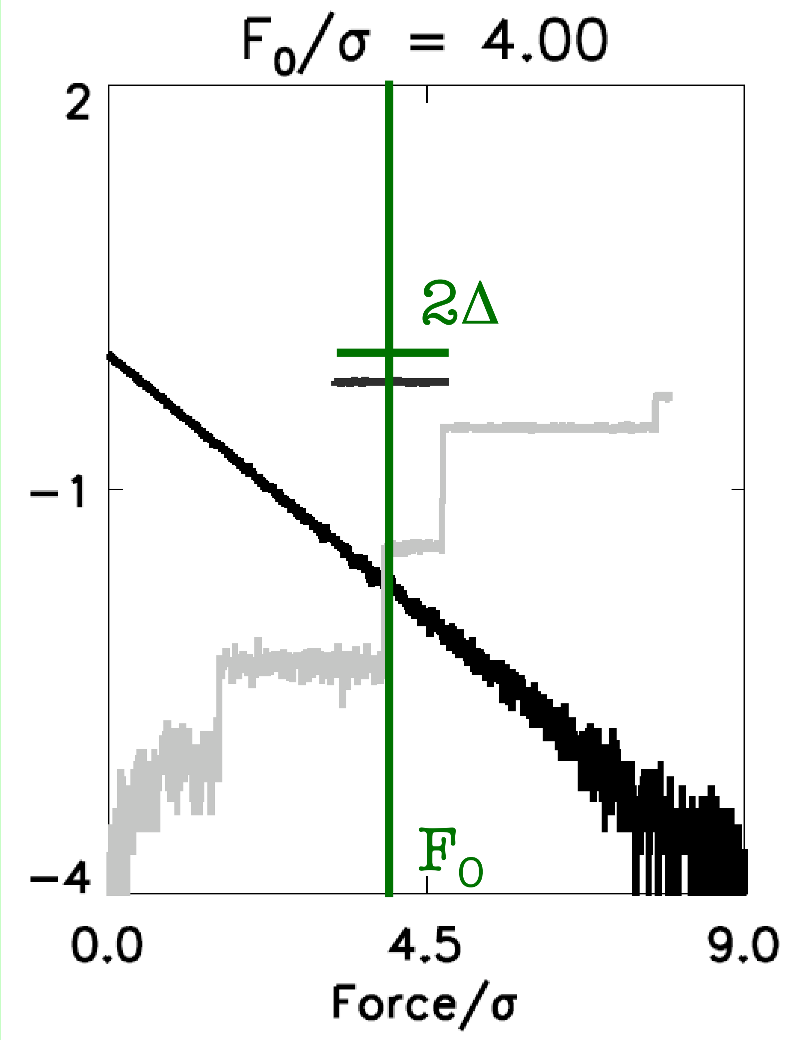
$$\begin{aligned} p(F_\mathrm{M}) &= e^{-F_\mathrm{M}/\sigma} \\ \sigma &\propto \dot{\nu}/\lambda \end{aligned}$$

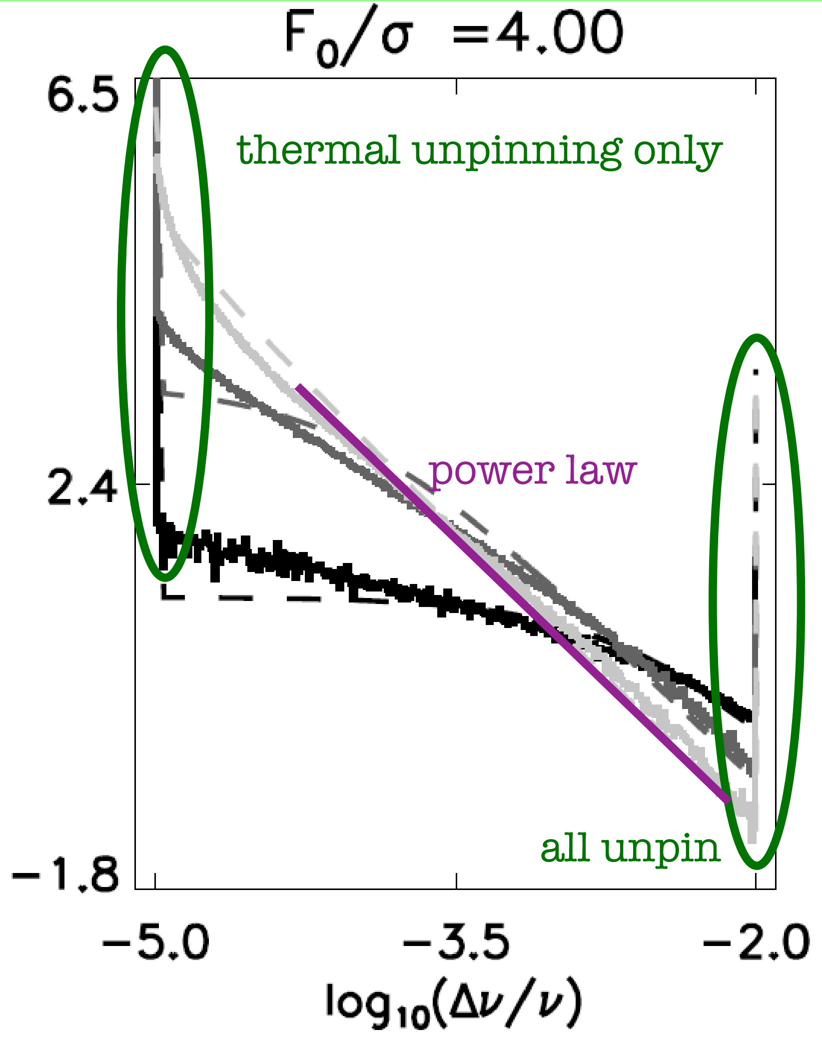
- Each pulsar has a different $p(F_M)$.

A schematic



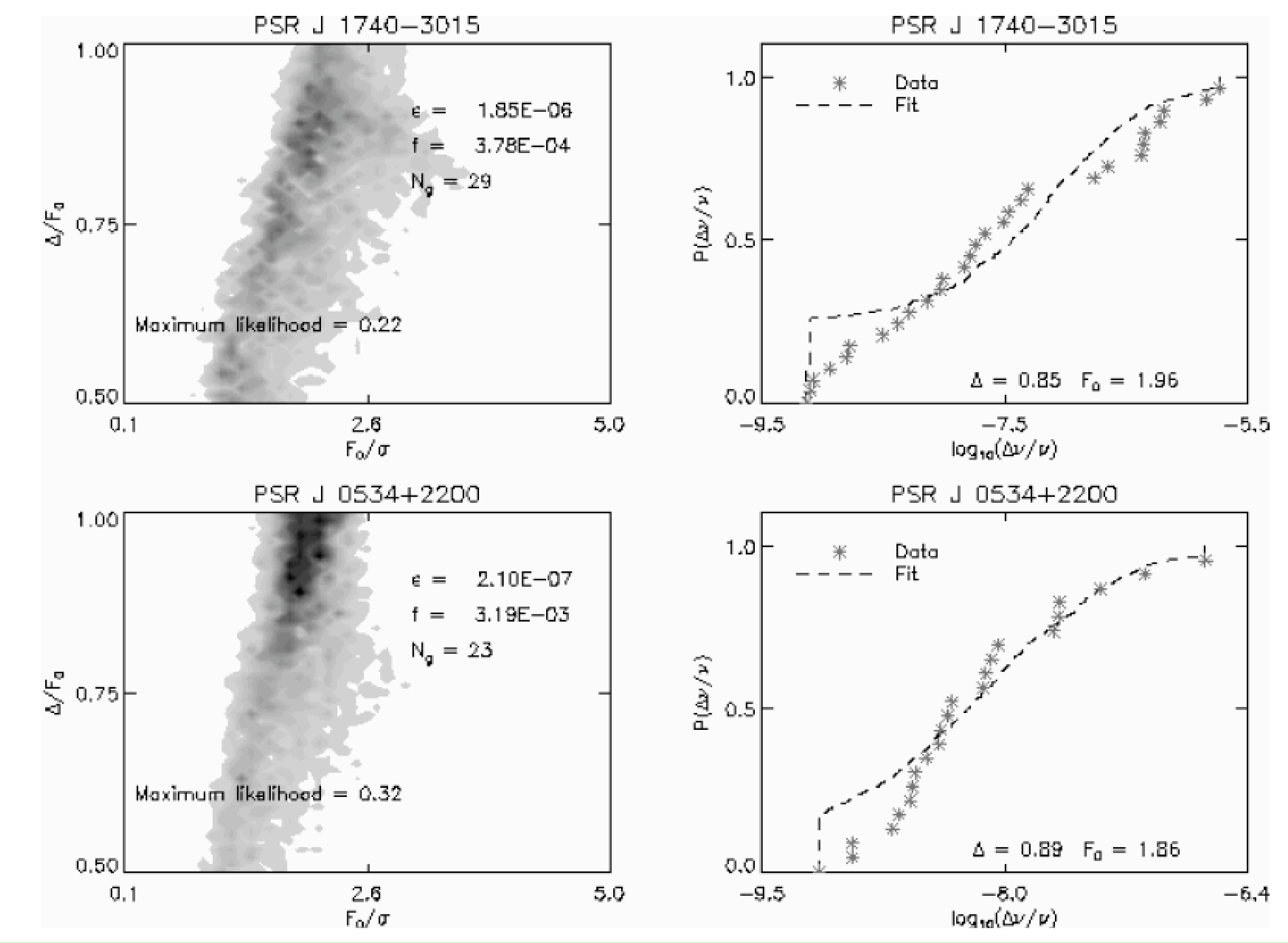
Computational output



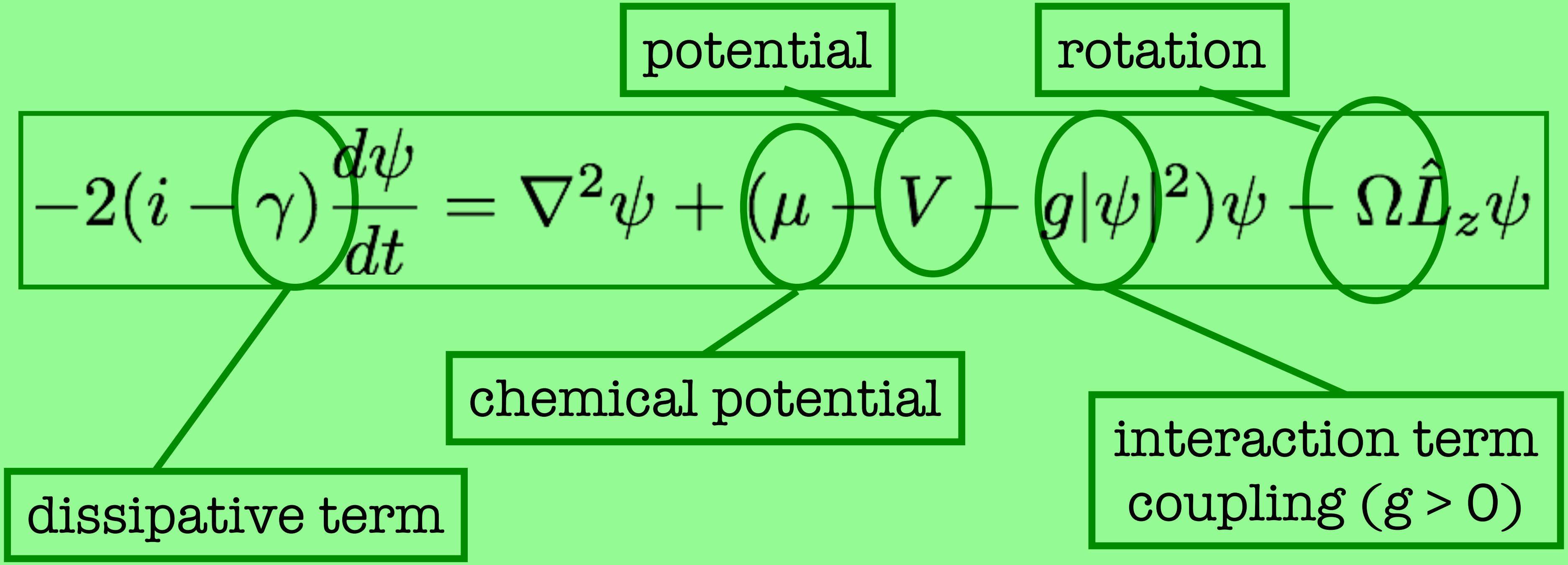


Model fits - Poissonian

- $F_0 \approx \Delta$ gives best fit in most cases.
- Broad pinning distribution agrees with theory: ≈ 2MeV ± 1MeV
- GW detection will make more precise



Gross-Pitaevskii equation



potential

rotation

$$-2(i-\gamma)\frac{d\psi}{dt} = \nabla^2\psi + (\mu - V - g|\psi|^2)\psi - \Omega\hat{L}_z\psi$$

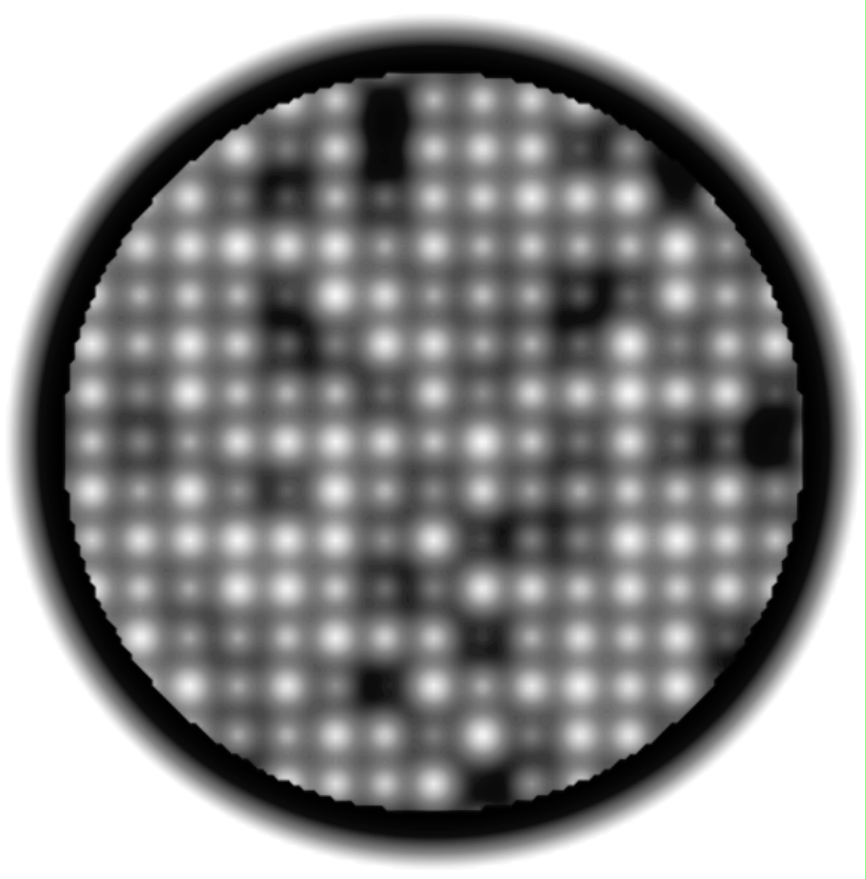
dissipative term

chemical potential

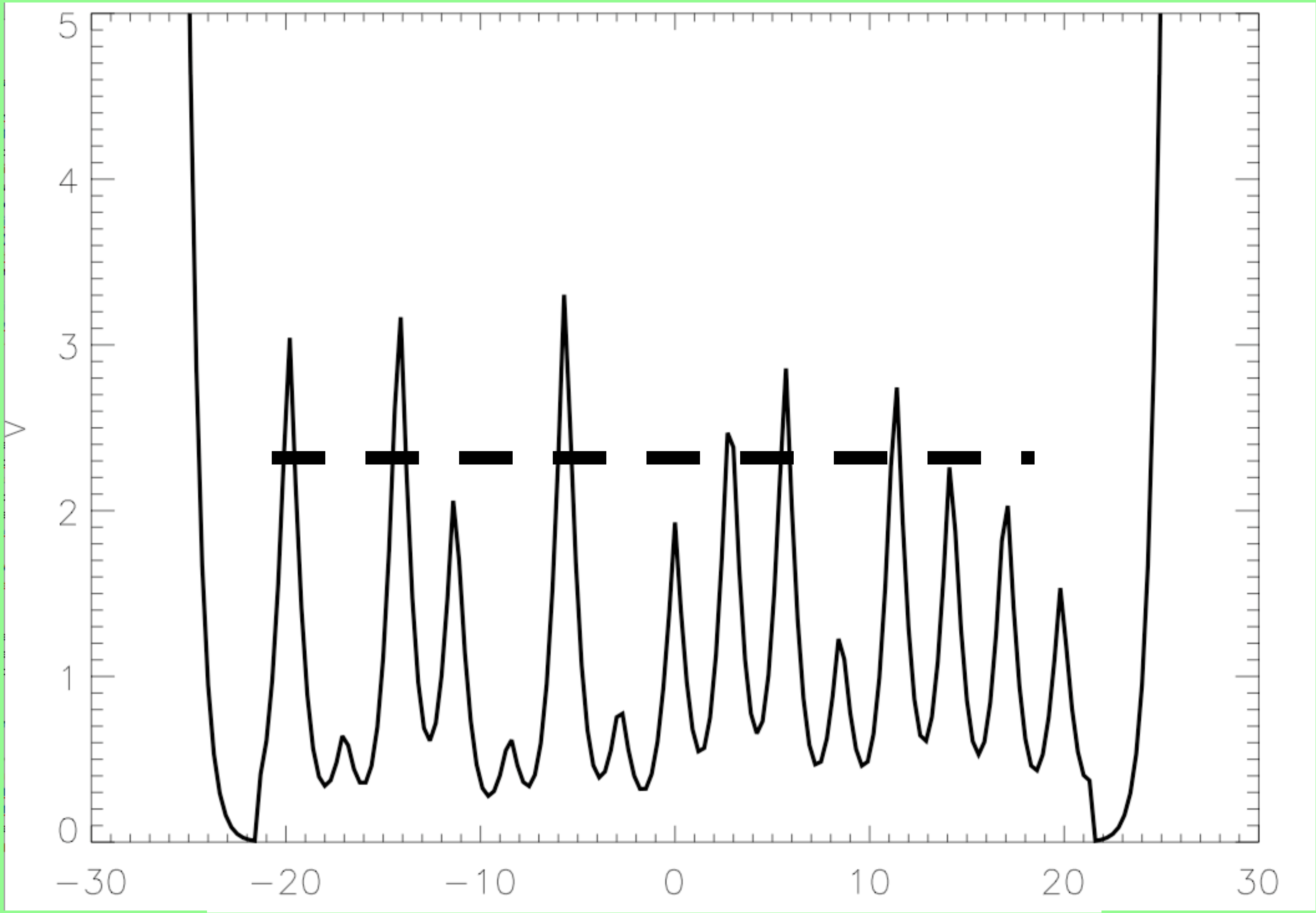
interaction term coupling ($g > 0$)

γ	(≡ 0.1)	suggests presence of normal fluid, aids convergence
V		grid of random pinning potentials
g	(≡ 1)	tunes repulsive interaction
μ	(≡ 1)	energy due to addition of a single particle
$\lvert\psi\rvert^2$		superfluid density

Spherical cows



The potential



Tracking the superfluid

$$\psi = \sqrt{\rho} e^{i\theta}$$

- Circulation counts number of vortices

$$\kappa N_{\mathrm{v}} = \int \mathbf{v} \cdot \mathbf{dl} \qquad\qquad \mathbf{v} = \nabla \theta_{\mathrm{phase}}$$

- Angular momentum L_z accounts for vortex positions

$$\langle \hat{L}_z \rangle = \int \underset{\underset{|\psi|^2}{\uparrow}}{\rho} \mathbf{x} \times \underset{\underset{\nabla\theta}{\uparrow}}{\mathbf{v}} d^3\mathbf{x}$$

Feedback equation

- Vortices move radially outward

  → superfluid slows down

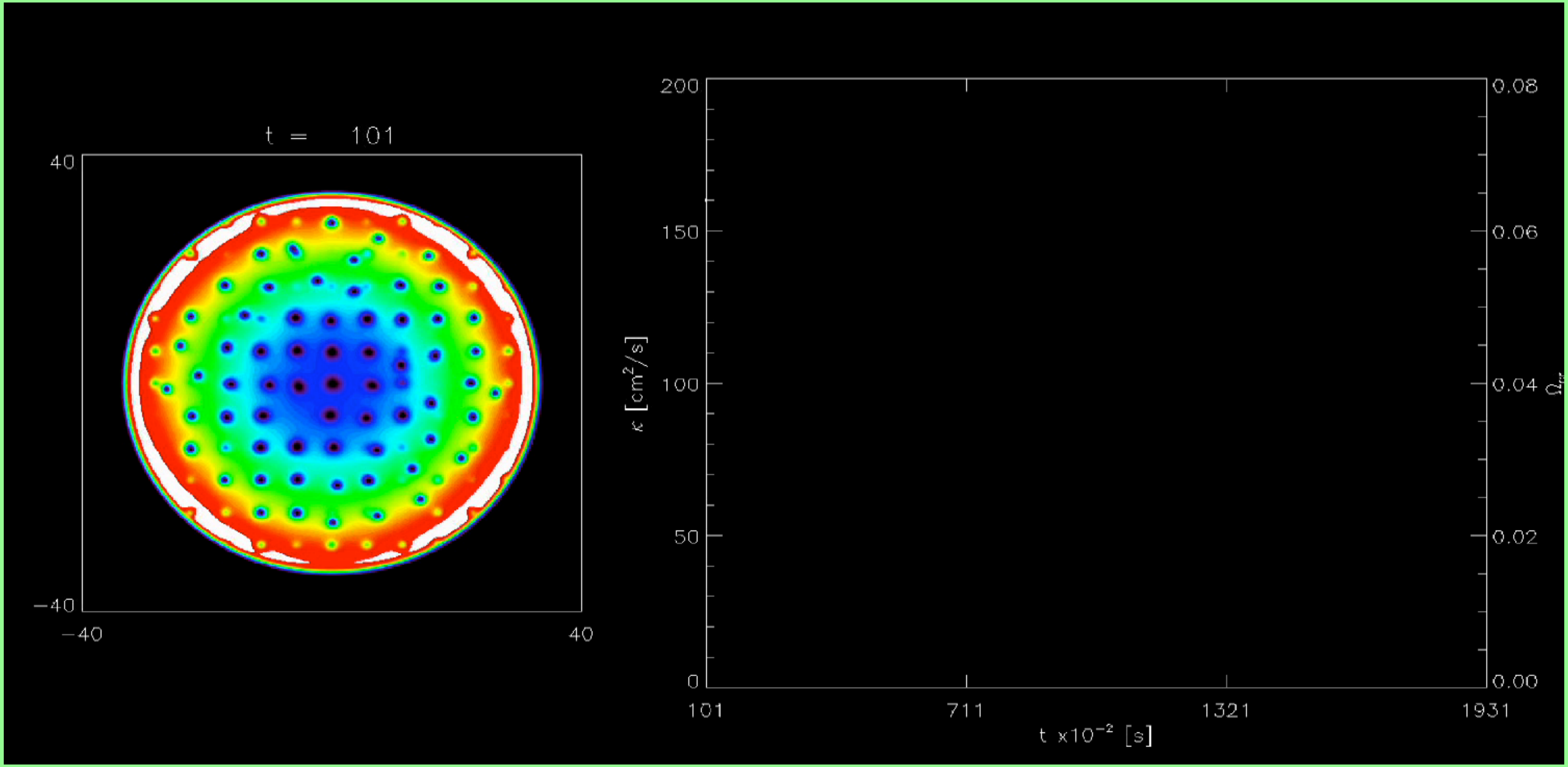
  → superfluid loses angular momentum

- Conservation of momentum: stellar crust gains angular momentum

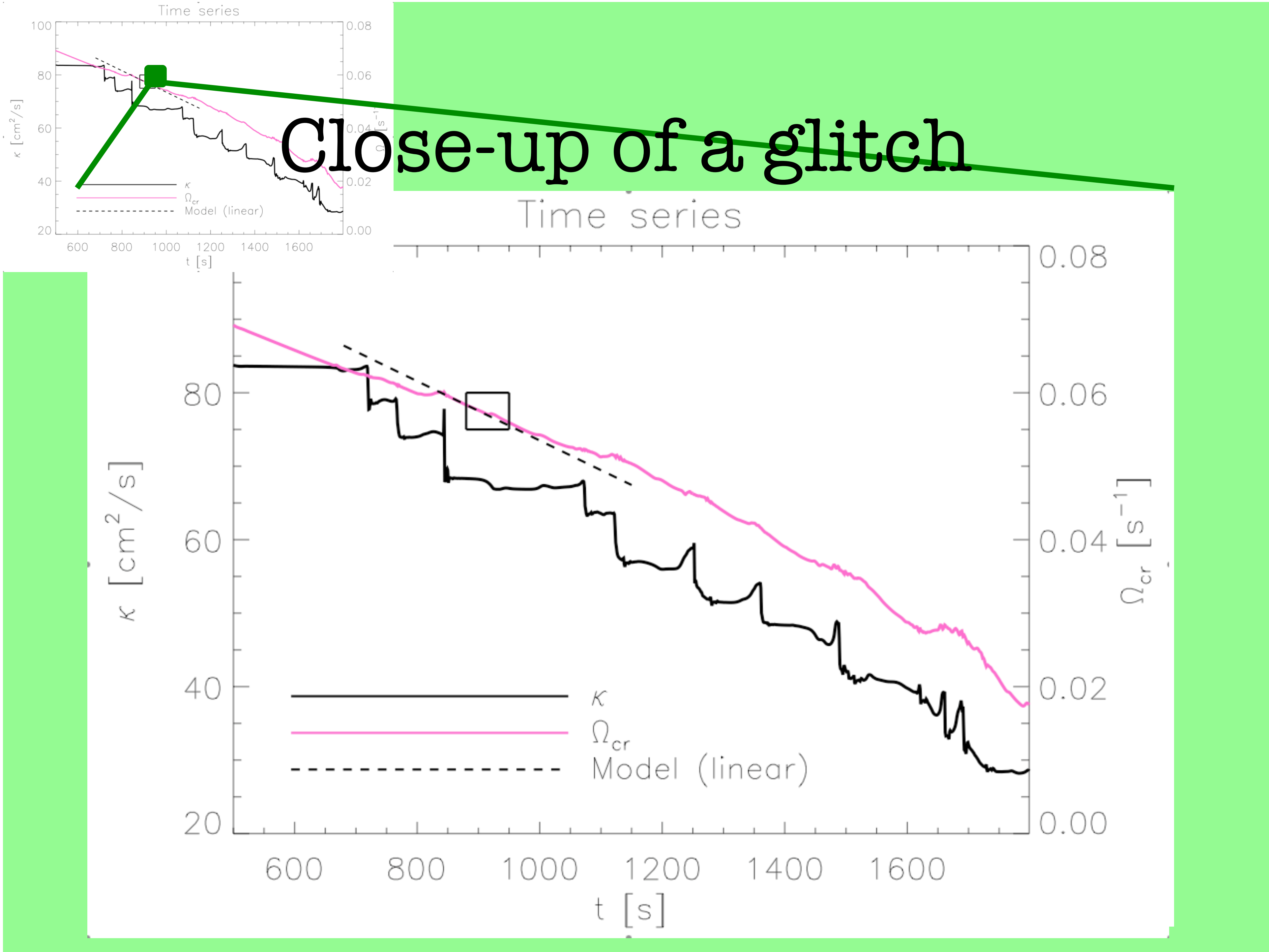
  → crust speeds up:

$$\boxed{\frac{d\nu_{\rm cr}}{dt} = -\frac{I_{\rm SF}}{I_{\rm cr}}\frac{d\nu_{\rm SF}}{dt} + \dot{\nu}_{\rm EM}}$$

Glitch simulations



Close-up of a glitch



Points to ponder…

- Glitch-like spin-up events do indeed occur.
- Evidence of correlations in vortex motion
  - Avalanches?
  - Coherent noise if collective behaviour strong enough
- Cannot make simulation large enough to get glitch statistics, but we're working on it…
- Ratio of pinning sites and vortices is far from the 'true' regime.
- Use individual characteristic vortex motion as Monte Carlo input.

Gravitational waves

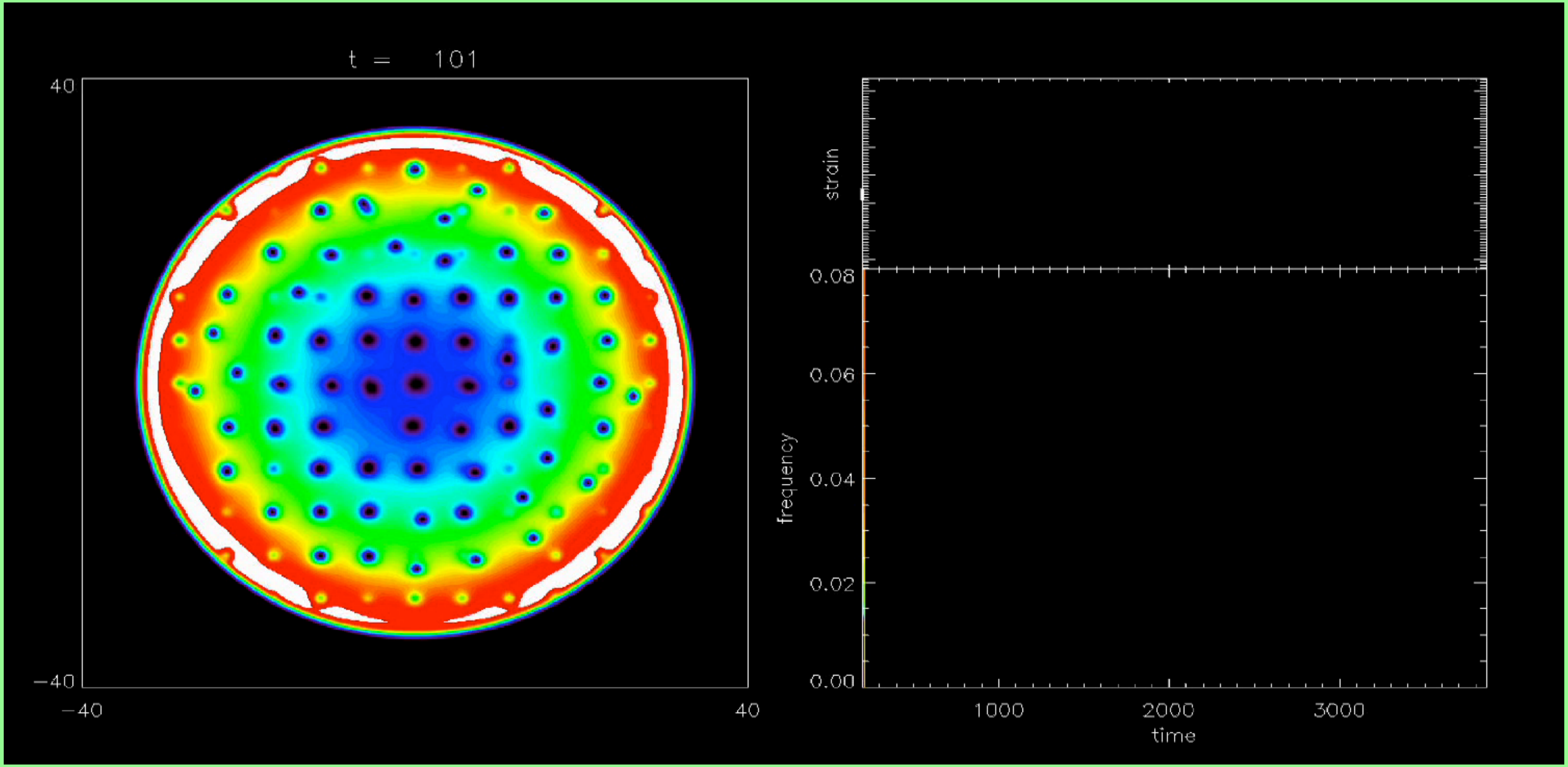
- Current quadrupole moment depends on velocity field

$$s^{lm} = c_l \int d^3x Y^*_{lm} r^l \mathbf{x} \cdot \nabla\times \ (\rho\mathbf{v})$$

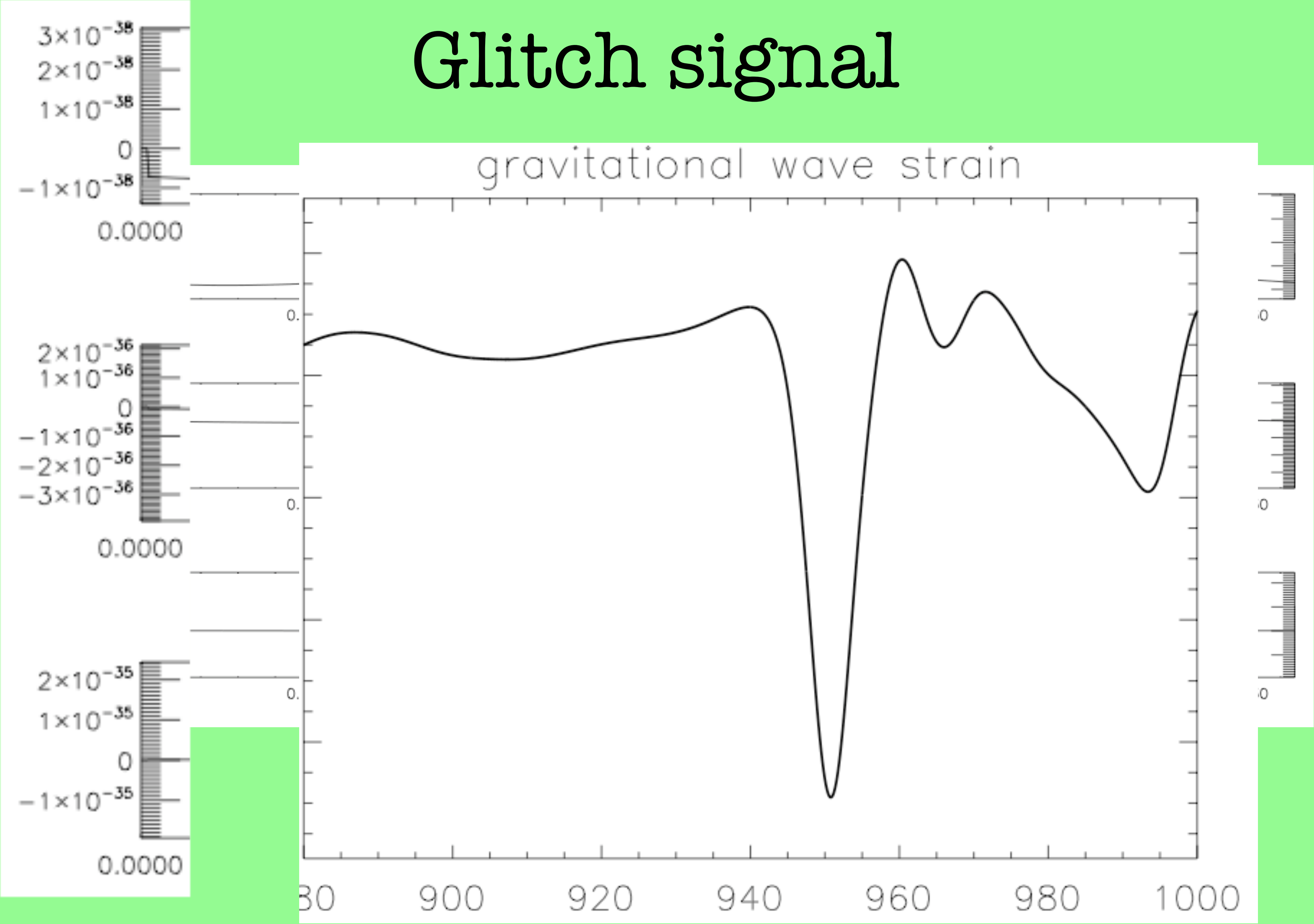
- Wave strain depends on time-varying current quadrupole

$$h^{\mathrm{TT}}_{jk} = \frac{G}{c^5 r} \frac{\partial^2 s^{21}}{\partial t^2} T^{\mathrm{B2,21}}_{jk}$$

Simulations with GWs



Glitch signal



Looking forward

- Wave strain scales as $\sqrt{N_{\mathrm{vort}}}$
  - Estimate strain from ‘real’ glitch:

$$h \approx 10^{-23} (\frac{\Delta t}{1\,\mathrm{ms}})^{-2} (\frac{N_{\mathrm{vortices}}}{10^{19}})^{1/2}$$

- First source?
  - Close neutron star (not necessarily pulsar)
  - Old, populous neutron stars ($\sim 10^8$)
  - Many pulsars aren’t timed - might be glitching
- Place limit on shear from turbulence [Melatos & Peralta (2009)]
- How to turn spectrogram into template appropriate to LIGO?
  - Incorporate new signals into LIGO pipelines.
  - Discriminate between burst types

What can we learn?

Nuclear physics laboratory <u>not possible on Earth</u>

- QCD equation of state (mass vs radius)
- **Compressibility:** soft or hard?
- State of superfluidity
- **Viscosity:** quantum lower bound?
- Lattice structure:
  - Type, depth & concentration of **defects**

Of interest to many diverse scientific communities!

Conclusions

- Many-pronged attack on the glitch problem motivated by **observed** pulsar glitch statistics.
- ‘Real’ glitch mechanism may be blend of avalanches, coherence and quantum effects.
- **First principles** simulations inform GW predictions.
- First calculation gravitational wave signal resulting from vortex rearrangement
  - detectable by LIGO?