LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T 0900161 4/9/2009

Coating Thermal Noise Formulas

Gregg Harry

Distribution of this document:
LIGO Science Collaboration

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project - MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW 22-295
185 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

The formula for coating thermal noise is

$$S_x(f) = 2k_B T \phi_{\text{eff}} \left(1-\sigma^2\right) / \left(\pi^{3/2} f w Y\right). \tag{1}$$

Here $S_x(f)$ is the power spectral density of position noise, k_B is Boltzmann's constant, T is the temperature, σ is the Poisson ratio of the substrate material, w is the half-width of the Gaussian laser beam, and ϕ_{eff} is the effective loss angle of the mirror. The loss angle can be written as

$$\begin{aligned} \phi_{\text{eff}} &= \phi + d/\left(\sqrt{\pi} w Y_\perp\right) \\ &\quad \left(\left(Y/\left(1-\sigma^2\right) - 2\sigma_\perp^2 Y Y_{||}/\left(Y_\perp\left(1-\sigma^2\right)\left(1-\sigma_{||}\right)\right)\right)\phi_\perp\right. \\ &\quad + Y_{||}\sigma_\perp(1-2\sigma)/\left(\left(1-\sigma_{||}\right)(1-\sigma)\right)\left(\phi_{||}-\phi_\perp\right) \\ &\quad \left.+ Y_{||}Y_\perp(1+\sigma)(1-2\sigma)^2/\left(Y\left(1-\sigma_{||}^2\right)(1-\sigma)\right)\phi_{||}\right), \end{aligned} \tag{2}$$

where d is the coating thickness, Y, σ, and ϕ are the Young's moduli, Poisson's ratios, and loss angles of the substrate (no subscript), and for the coating for stresses perpendicular ($_\perp$) and parallel ($_{||}$) to the optic face.

The elastic constants of the coating can be calculated from the bulk values of the individual materials that make up the coating (silica and titania doped tantala in the requested LIGO mirrors). For a coating made of alternating layers of two dielectrics, these values are

$$\begin{aligned} Y_\perp &= (d_{ta}+d_{si})/(d_{ta}/Y_{ta}+d_{si}/Y_{si}) && (3)\\ Y_{||} &= (Y_{ta}d_{ta}+Y_{si}d_{si})/(d_{ta}+d_{si}) && (4)\\ \sigma_\perp &= (\sigma_{ta}Y_{ta}d_{ta}+\sigma_{si}Y_{si}d_{si})/(Y_{ta}d_{ta}+Y_{si}d_{si}) && (5)\\ \phi_\perp &= Y_\perp/(d_{ta}+d_{si})(\phi_{ta}d_{ta}/Y_{ta}+\phi_{si}d_{si}/Y_{si}) && (6)\\ \phi_{||} &= (Y_{ta}\phi_{ta}d_{ta}+Y_{si}\phi_{si}d_{si})/\left(Y_{||}(d_{ta}+d_{si})\right), && (7) \end{aligned}$$

where subscripts ta and si refer to the titania doped tantala and silica coating materials respectively. For $\sigma_{||}$ the equivalent equation is more complicated. Averaging the two material Poisson ratios agrees with a numerical solution to within 5% for the case of a silica/titania doped tantala coating with quarter wave layers for high reflectivity at 1.064 μm, the laser wavelength.

Appropriate numerical values to be used in these equations are

$$\begin{aligned} T &= 290\text{ K} && (8)\\ f &= 100\text{ Hz} && (9)\\ w &= 6\text{ cm} && (10)\\ Y &= 72.7\times10^9\text{ Pa} && (11)\\ Y_{ta} &= 140\times10^9\text{ Pa} && (12)\\ Y_{si} &= 72.7\times10^9\text{ Pa} && (13)\\ \phi &= 10^{-8}\text{ rad} && (14)\\ \phi_{ta} &= 2.3\times10^{-4}\text{ rad} && (15)\\ \phi_{si} &= 4.0\times10^{-5}\text{ rad} && (16)\\ \sigma &= 0.17 && (17)\\ \sigma_{ta} &= 0.23 && (18)\\ \sigma_{si} &= 0.17 && (19) \end{aligned}$$