LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
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Tip-Tilt Mirror Specifications and Design

Bram J. J. Slagmolen, Adam Mullavey,
Rana Adhikari, David E. McClelland

Distribution of this document:

Detector Group

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW17-161
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

http://www.ligo.caltech.edu/

1 Introduction

The GW interferometers are becoming more optically complex and advanced. In the planned upgrade of the initial LIGO (iLIGO) interferometers with Advanced LIGO technologies, i.e. enhanced LIGO (eLIGO), a DC readout scheme is employed. This scheme utilizes an output mode cleaner (OMC) to filter out so called junk light in the interferometer output beam before the DC signal readout []. The OMC is a bow tie optical cavity, consisting of four mirrors which are glued to a glass breadboard. This breadboard is suspended upside down from an double pendulum suspension stage. One of the cavity mirrors is mounted on a PZT for fast, short range actuation of the cavity length. Another cavity mirror is mounted on a thermally actuated aluminium tube for slow, long range actuation.

For the active beam steering, we need a vacuum-compatible mirror mount that has suitable range and bandwidth. Some vibration isolation is also desirable to reduce scattered-light path length noise. It is hard to know how much range is really needed, but probably at least a few milliradians. The bandwidth required for beam pointing stabilization is expected to be tens of hertz or less. On the other hand, we intend to generate alignment signals for the OMC via dither of the beam direction; this small-amplitude dither should be at as high a frequency as possible.

One option for the mount is a commercial PZT-based tip-tilt mirror, such as the type used in the iLIGO beam line readout system. However, these units have some disadvantages: their range is small (in order of a few milliradians); in our experience they are not very robust; they offer no vibration isolation. Given these downsides, we decided to develop our own mount instead. The design concept for these is something between the Newport Fast-Steering Mirror and the iLIGO small-optics suspension (SOS). The design, called a tip-tilt mount, has been developed and tested at ANU, and is being implemented on the eLIGO OMC beamline.

2 Design Motivation

Behind the signal recycling mirror, the beam will be guided towards the OMC. Depending on the signal recycling cavity topology (non-stable, stable), this may included a standard beam reducing telescope, prior the mode-matching into the cavity. The mode matching into the OMC will be done by two mode matching mirrors, each mounted into a tip-tilt steering mount. The tip-tilt mirrors will be responsible for the alignment of the interferometer beam into the OMC.

The OMC auto-aligment system will be done using dither locking. That is, the tip-tilt steering mirrors will be dithered in pitch and yaw in the kHz regime to create an error signal in the DC readout of the OMC transmitted beam. In effect the dither will slightly misalign the beam incident on the OMC, creating a power fluctuation at the dither frequency. This power fluctuation on the DC photodiodes will be demodulated to obtain an error signal for the appropriate degree of freedom. This will be done for all four degrees of freedom (pitch and yaw for in the near-field and far-field).

The dither angle for the tip-tilt can be obtained using the OMC transmitted power fluctuations which can be described by:

$$\Delta P = P_0\left(1 - 2\left(\frac{x}{\omega_0}\right)^2 - 2\left(\frac{\theta}{\theta_0}\right)^2\right) \tag{1}$$

where x is the lateral beam displacement and θ the beam tilt at the OMC waist, and ω_0 and θ_0 are the OMC waist size and divergence angle.

The dither signals on the DC photodiodes should be strong enough, but not dominate the DC photodiodes signals. From figure 10, the maximum dithar angle at 1 kHz is approx. 7μrad.

Rewriting the equation 1 into tilt dependent relative intensity noise,

$$\frac{\Delta P}{P_0} = 2\left(\frac{\theta}{\theta_0}\right)^2 \tag{2}$$

With an OMC waist $\omega_0 = 477\,\mu$m and a divergence angle $\theta_0 = 710\,\mu$rad, the normalised power fluctuation is $2 \cdot 10^{-4}$. This is about 5 orders of magnitude above the required laser RIN.

The sensing noise of the dither signal on the DC photodiodes depend on the photo current shot noise level. The photo current in the photodiode goes linear with the incident power. Rewriting equation 2 and replace θ/θ_0 by u, and substitute power P with current I, the current fluctuations, can be described by,

$$\Delta I = 2I_{dc}u^2 \tag{3}$$

The slope (or gain sensitivity) can be written as:

$$\frac{dI}{du} = 4I_{dc}u \tag{4}$$

Setting the DC photo current to 100 mA, the shot noise dI will be at $1.8 \cdot 10^{-10}$ A/$\sqrt{\text{Hz}}$. Then setting $u = \delta$ as the relative amplitude fluctuations, the shot noise sensitivity du, $1/\sqrt{\text{Hz}}$, is given by:

$$du = \frac{4.5 \cdot 10^{-10}}{\delta} \tag{5}$$

Setting $\delta = 10^{-2}$ and $du = \delta x/\omega_0$, the displacement sensitivity at the OMC waist is $\sim 2.2 \cdot 10^{-11}$ m/$\sqrt{\text{Hz}}$. This approach provides a general shot noise sensitivity per relative amplitude fluctuation (or dither strength), due to lateral offset or angular tilt.

3 Design Requirements

There are a few design requirements the Tip-Tilts will need to adhere to, and are listed below:

1. fundamental resonances above 2 Hz,
2. dither ability at 1 kHz (using the BOSEMs),
3. residual POS motion, when placed on HAM ISI, below scatter limit,
4. vertical bounce mode close to 343 Hz (to coincide with the LOS bounce mode).

4 Design Concept

The main design requirement for the Tip-Tilt Mirror was to make a 2-inch mirror which would be able to dither in tip and tilt at ∼1 kHz, with an angular deviation of a few μrad. The design is based on a single loop wire suspension system. The actual mirror is mounted inside a metal ring to which the suspension wires are clamped. This mirror assembly is suspended by two separate wires of equal length which are clamped on each side. Also attached to this metal ring are four magnets, which are used in conjunction with four coils for the control and actuation of the angular and positional degrees of freedom (DOFs).

All of the resonant frequencies are designed to be above 2 Hz. The Tip-Tilt Mirror will be mounted on top of an isolated platform, which has resonances up to ∼ 2 Hz. Although the isolated platform will be controlled, to prevent damage to the mirror in the situation that these controls fail, the table resonance will not overly excite the Tip-Tilt resonances.

To reach the angular dither requirement the inertia of the suspended mirror is minimised. This is achieved by using an aluminium ring in which the actual optical mirror is mounted. In addition, the metal ring provides the possibility to use screws instead of glue to mount the magnets and wire clamps.

The design has been modeled using the TwoWireSimple pendulum Mathematica notebook developed by M. Barton for the LIGO project [2].

5 Mechanical Design

The mass of the mirror assembly, including magnets and clamps, is 124 g. The inertia of the mirror assembly is calculated to be $\sim 50\,\mathrm{kg{\cdot}m^2}$. The magnets are located on a circle with a radius of 34.1 mm from the center of the optic.

The two suspension wires have a length of 5.5 mm which makes the main pendulum resonance 2.21 Hz. The pitch resonance depends on the position the suspension wire leaves the optic in respect to the centre of mass (c.o.m) of the mirror assembly. This is set to 1.1 mm above the c.o.m. resulting in a pitch resonance of 4.98 Hz. The yaw resonance is controlled by the separation of the suspension wires at the suspension point in respect to the effective diameter of the optic. This is set to 60 mm at the suspension point and 76.2 mm at the optic, resulting in a yaw resonance of 4.63 Hz. Figures 2. 3 and 4 show the modeled, measured and fitted transfer functions of the POS, PIT and YAW degrees of freedom. Only the amplitude offset has been adjusted to best fit the measured response. The fitted response can be used in the real-time EPICS feedback control for the Tip-Tilt Mirrors.

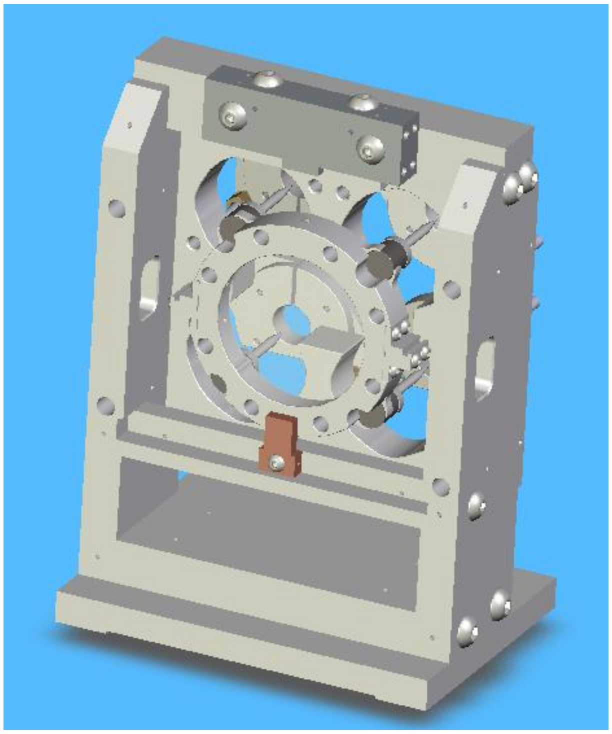

Figure 1: Drawing of the Tip-Tilt Stage.

Table 1: The measured and modeled resonance frequencies of the Tip-Tilt Stage.

Mode	Modeled	Measured	Deviation
Pendulum	2.21	2.27	3%
Pitch	4.98	5.35	7%
Yaw	4.63	5.46	15%
Side	3.09	3.07	0.6%
Vertical	356	?	?
Roll	xxx	yyy	?

The vertical bounce mode has been made to coincide, as much as possible, with the LIGO core optics violin mode resonance at 343 Hz. This is achieved by using a suspension wire of 380 μm. All the major first order resonance frequencies are shown in table 1, while figure 1 shows the Tip-Tilt design drawing.

To be able to optimise the DC pitch offset during assembly, the suspension wires are clamped onto little ears, which can slide back and forth in respect to the optic assembly. Once adjusted, they can be screwed into place.

The mirror assembly is being actuated on by an Optical Sensor and Electro-Magnetic actuator combined in a single unit (OSEM). Within an OSEM a LED and a photodiode pair operate as a shadow sensor. A flag is mounted on the back of each magnet and positioned in the LED beam so that one half of the LED beam is incident on the photodiode. Any positional fluctuations of the flag will be registered in the photodiode readout. A coil and

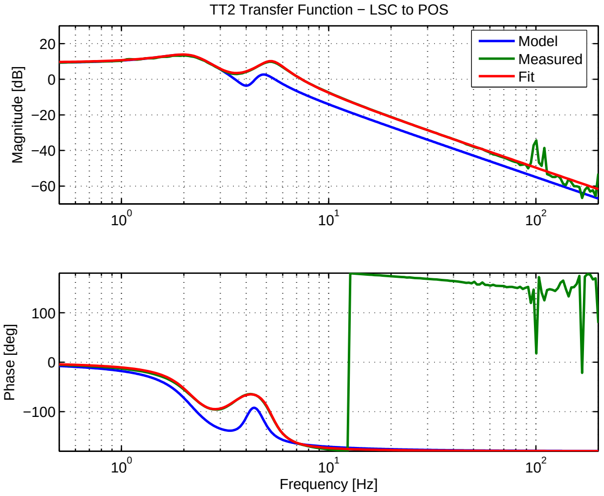

Figure 2: Transfer function of the Tip-Tilt mirror in position.

magnet pair act as an actuator, when controlling the current through the coil.

Originally the BOSEMs have a dynamic range of 0.7 mm. This range is extended to ∼4 mmb y using a pointed flag, which has a 1:6 ratio tip [].

The OSEMs are held by the backplane which has been designed to minimise the eddy current damping (ECD) of the OSEMs. In the current design the ECD due to the OSEMs dominates the resonance quality factor and transfer function response.

In the design a single OSEM provides an ECD of $\sim$ 0.37 kg/m^2, with four OSEMs this results in quality factors for the resonances of $2 - 3$. Although this provides an automatic damping effect, it also modifies the isolation response above the resonance frequencies (e.g. $>$ 10 Hz) from $1/f^2$ to $1/f$.

With four OSEMs, only three DOFs can be controlled (pendulum, pitch and yaw). The other three DOFs - side, roll and vertical, are to be damped by using ECD with a magnet and a piece of copper located at the front of the mirror.

To be able to obtain the dither signal at a kHz, we opted for using four Birmingham Noise Prototype OSEMs[1]. These OSEMs have a displacement sensitivity of 100pm/$\sqrt{\text{Hz}}$ above 1 Hz [1], and in combination with large NEO magnets (10 mm diameter x 10 mm length), can achieve a force coefficient of 2.05 N/A. To prevent damage and vacuum compatibility problems from the out-gassing due to the generated heat, it is required that the current through the coils of the OSEMs does not exceed 100 mA. As the mirrors are to be installed inside the high vacuum environment, a technical design requirement for the steering mirrors is to be vacuum compatible.

5.1 LSC/POS Compliance

The POS (or longitudinal position) of the Tip-Tilt need to be below the scattered surface displacement as described in T060303 by P. Fritschel. For AdvLIGO the maximum displace-

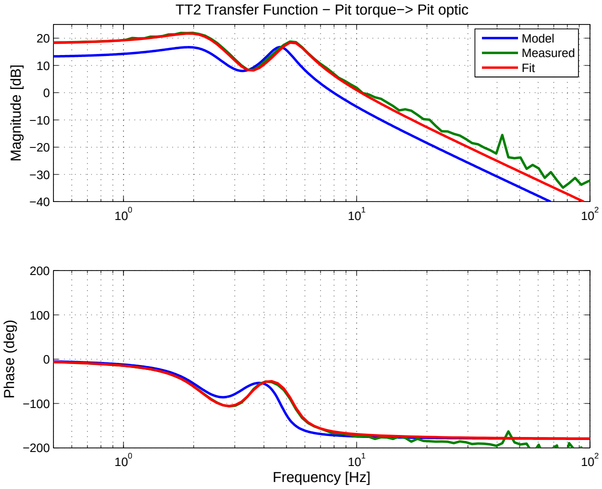

Figure 3: Transfer function of the Tip-Tilt mirror in pitch.

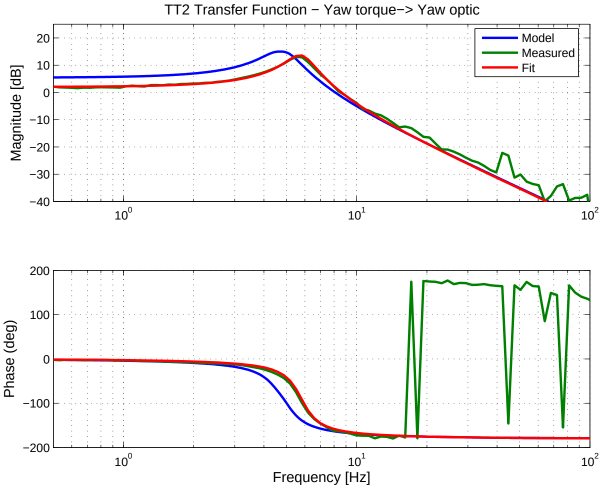

Figure 4: Transfer function of the Tip-Tilt mirror in yaw.

ment of a scattered surface between the SRM and output mode cleaner is given by,

$$x_{sc} < 10^{-17}/\sqrt{R_r} \qquad \mathrm{m}/\sqrt{\mathrm{Hz}}, f > \ 10 \text{ Hz} \tag{6}$$

and this included a safety factor of 10. The value R_r is approximately $2 \cdot 10^{-13}$ for a CVI mirror, so $x_{sc} < 2.2 \cdot 10^{-11}$ m/$\sqrt{\text{Hz}}$ for frequencies above 10 Hz.

Figure 5 shows the displacement limit of a CVI mirror in the beam line between the SRM and the output mode cleaner. The response below 10 Hz has been amended with an $1/f^2$ slope. The residual HAM ISI displacement with active feedback from the witness sensors has been used to make the Tip-Tilt displacement in the X, Y and Z dimensions. The vertical Z-dimension is the coupling from the vertical motion of the Tip-Tilt suspension point to the mirror motion into POS, due to the thick 380 μm diameter wires, this is very stiff.

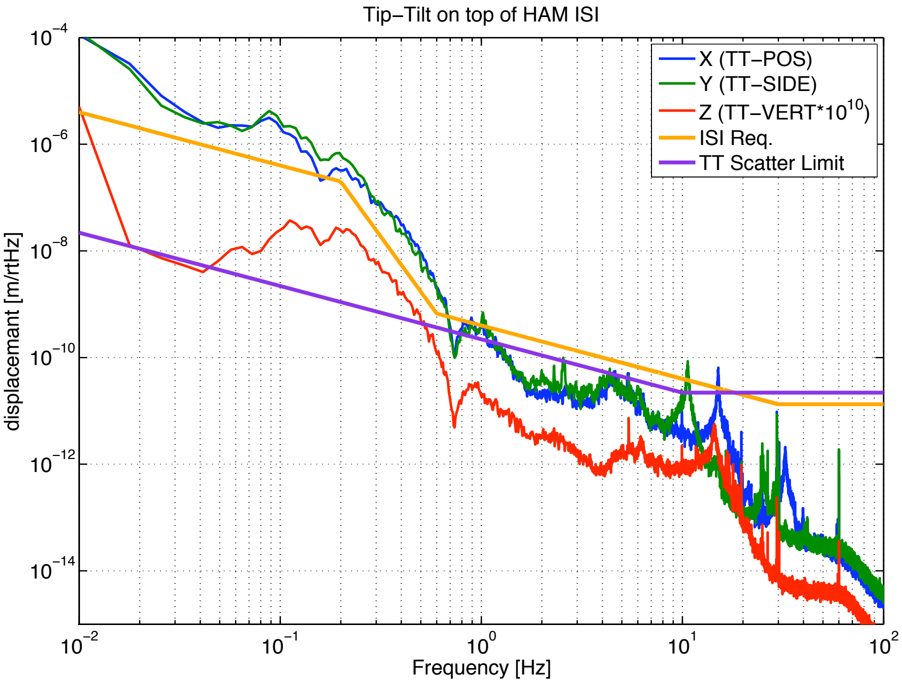

Figure 5: Tip-Tilt residual motion when placed on top of a HAM ISI with active feedback. The curve labeled 'TT Scatter Limit' is obtianed with eqn. 6, and follows as $1/f^2$ at frequencies below 10 Hz.

Table 2: Internal resonant modes of the major part of the Tip-Tilt mirror.

Mode	Structure	Mirror Holder	Flag
1	636	4818	2911
2	1017	4905	2913
3	1567	8192	16834
4	2755	8288	16856
5	3049	13294	16856

5.2 Structural Simulation

To investigate what the mechanical structural resonances are, the Solidworks® assembly drawings have been simulated using COSMOS finite element analyses. The first five resonances are listed in Table 2. The support structure has been simulated and the first resonance has been modeled to be 636.5 Hz. Also the mirror holder has been modeled, as well as the flag attached to the four magnets on the mirror holder.

5.3 Possible Design Alterations

There are a few design changes which make the Tip-Tilt useful in other configurations. Or to recover the $1/f^2$ frequency response above the resonance.

5.3.1 Passive Tip-Tilt

Due to the low effective quality factors of the resonance modes, it is perceived that the tip-tilt mounts can be used for passive steering/folding mirrors. The active OSEMs are then replaced with eddy current dampers for the resonant modes. To overcome the $1/f$ response above the resonance frequency due to the ECD, the dampers are being suspended as well. This can be achieved by mounting the dampers on a flexure which has a resonance frequency above that of the tip-tilt mirror pendulum resonance [3]. With such an assembly the $1/f^2$ response will be restored at frequencies above twice the flexure resonance. This is because below the flexure resonance frequency, the response between the damper and the tip-tilt mirror is flat and the ECD is effective (which has an f-dependency). Above the flexure resonance frequency the $1/f^2$ response of the flexure will modify the effective ECD to a $1/f$ response. Combined with the tip-tilt pendulum response of $1/f^2$ the total ECD response will drop as $1/f^3$ and go under the $1/f^2$ tip-tilt response, at which point the original $1/f^2$ response is restored.

5.3.2 BOSEM ECD reduction

The $1/f$ isolation response above 10 Hz of the tip-tilts with active OSEMs will be sufficient in Enhanced LIGO. It is possible to restore the $1/f^2$ isolation response in case increased isolation is required. This can be achieved by replacing the aluminium backplane in which the OSEMs are held by a non-conductive material such as PEEK. The drawback of using a non-conductive backplane is that the OSEMs need to be used to damp the optic as the ECD effect has been removed.

5.3.3 Lighter Mirror Holder

Another alteration in the design is to make the mirror holder lighter. This can be done by lighten the aluminium ring differently or making it from PEEK. Making it from Titanium makes it twice as strong, but ∼twice as heavy. Although 7075 aluminium would be another option, in that case a LIGO vacuum waiver is required as currently 7000 series aluminium is prohibited inside the LIGO vacuum envelope.

6 Performance

Two key performances are to be met, the damping performance of the side and vertical bounce modes using ECD and the high frequency dither. Due to the strong ECD from the OSEMs in the design and the side ECD, the ECD will dominate the performance at frequencies around the resonance frequencies ($<$ 30Hz). Figure 7 shows the power spectrum of a free swinging Tip-Tilt Stage, with and without the lower-right OSEM. By fitting a second order function to the main pendulum response (near 2.2 Hz), we can establish the amount of damping one OSEM provides.

Figure 6 shows the calculated longitudinal transmissibility, obtained using the measured effective longitudinal damping and quality factors (from Figure 7). The graph indicates a

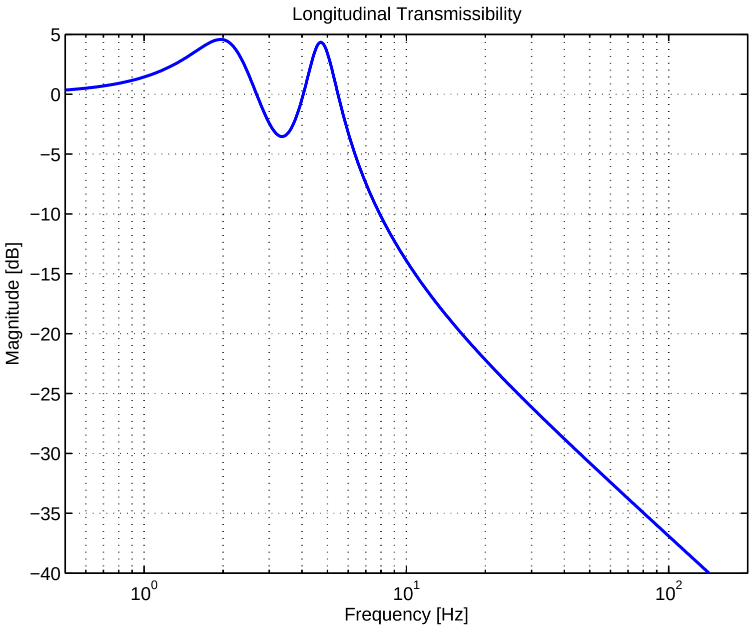

Figure 6: The calculated longitudinal transmissibility of the the Tip-Tilt, which at the resonant frequency of 2.26 Hz is 1.27. Note that due to the dominance of the Eddy Current Damping the slope above resonance is $1/f$, instead of $1/f^2$.

strong damping and a $1/f$ isolation response above the resonance frequency.

Table 3 shows some basic properties of the Tip-Tilts.

Table 3: Baisc mechanical and optical properties of the Tip-Tilt.

Angular dynamic range	±17 mrad
POS dynamic range	∼4 mm
Optics size	50.8 mm x 10 mm (2-inch x 3/8-inch)
Transmission aperture (diameter, centred)	28 mm vertical ∼ 35 mm horizontal
Beam height	101.6 mm (4-inch)
Suspension wire diameter	381 μm

6.1 Angular Dither

The dither performance is measured by a laser incident onto the Tip-Tilt Mirror and directed onto a quadrant photodiode (QPD). The quadrants are organised, such that the detector provides a sum, x-, and y-output which are digitised. The beam on the detector is aligned onto the center of the quadrants, apart from the sum output, all signal level are minimized. Reading out the x- and y-output of the QPD provide an indication of the tip and tilt of the mirror. The transfer function of the angular pitch motion to the y-output of the QPD

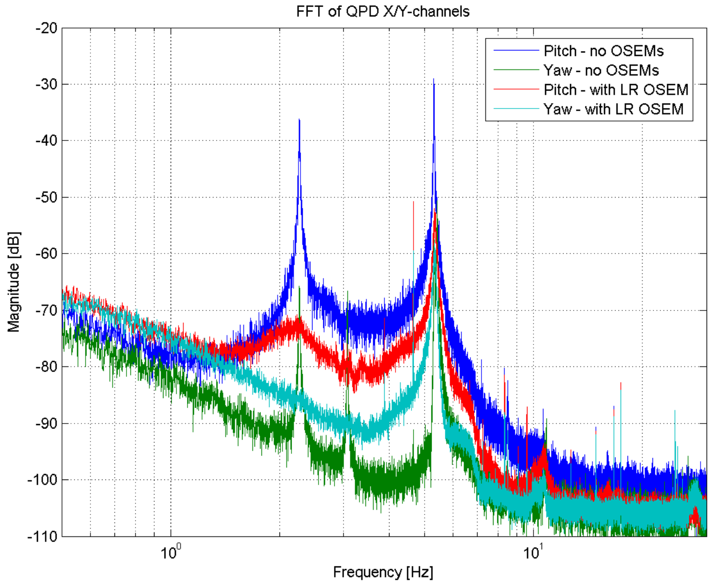

Figure 7: Power spectrum of the X- and Y-output of the QPD with and without the LR OSEM, indicating the amount of damping the OSEM provides.

is measured, and shown in figure 8. At the same time the x-output is recorded and shown. The figure indicates that dither frequencies of up to ~ 2 kHz are achievable, without running into mechanical resonances.

Around 3 kHz are the OSEM flag resonances (cantilever beam resonance). Then just below 5 kHz are the first two mirror holder resonances. The peak at 630 Hz in the lower trace is the first bending mode of the strucure.

7 Conclusion

We have designed, built and tested a new Tip-Tilt Mirror following some initial design requirements. The main dither requirements is met by a factor of two. Although the eddy current damping in the design may be large, they provide adequate damping around the resonance frequencies. All natural frequencies are above ∼2 Hz, while the vertical mode is close to the LIGO bounce mode of 343 Hz.

We would like to thank R. Adhikari and P. Fritschel for their advice during the design and tests.

A Design Considerations

1. TT position TF, due to ISI table at 10 Hz must be below design. Using T060303 $\rightarrow x_{sc} < 10^{-17}/\sqrt{R_r}$. $R_r \approx 2 \cdot 10^{-13}$ for a CVI mirror. So $x_{sc} < 2.2 \cdot 10^{-11}$ m/rtHz, using ISI noise curve at 10 Hz $\approx 4 \cdot 10^{-11}$ m/rtHz, and with a TT transmissibility at

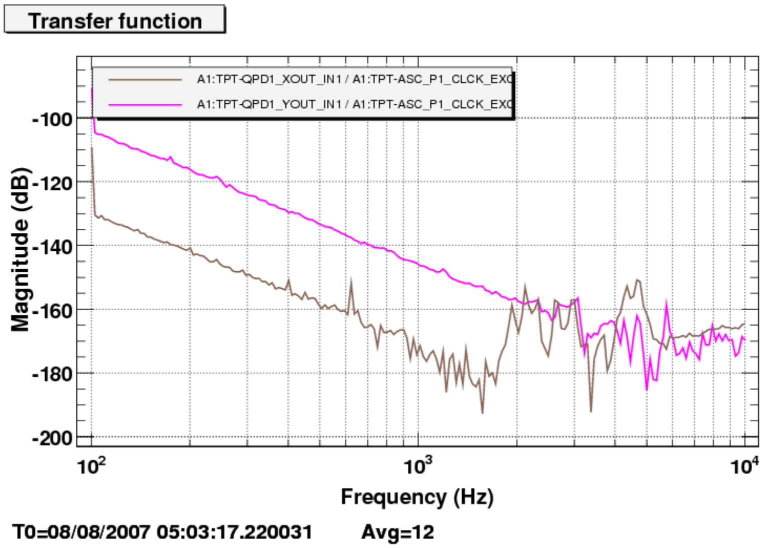

Figure 8: Dither transfer function of the Tip-Tilt. The mirror is driven in pitch, while the X- and Y- outputs of the QPD are recorded.

   10 Hz of 0.177, the $x_{sc,TT} = 7 \cdot 10^{-12}$ m/rtHz.

2. Nonlinear upconversion via the hysteresis in the wires. We know that there is hysteresis when we put DC biases so there must be either upconversion or downconversion. But does this matter? Depends on the ISI spectrum.
3. What about vertical compliance? Do we need some? Where should the vertical mode frequency be? Does the vertical noise couple through the angular noise and/or the coupling to the longitudinal?
4. How to get rid of eddy current damping without screwing up the stiffness of the thing?
5. If we don't need to dither these guys so hard can we reduce the magnet size? Do we have to reduce the magnet size to avoid coupling from the ISI coil drivers?
6. passive TT, these units can use the ECD mounted on a flexure.
7. When changing backplane make sure that the backplane is not absorbing (damping) the reaction force.

References

[1] S. M. Aston and D. M. Hoyland, *Noise prototype* OSEM *design document & test report*, Tech. Report LIGO-T050111-02-K, LIGO, 2008.

[2] Mark A. Barton, M*ark* B*arton's* S*uspension* M*odels* P*age* - LIGO-I, T*wo*-W*ire* S*imple* P*endulum*, (2008), http://www.ligo.caltech.edu/~e2e/SUSmodels/index.html.

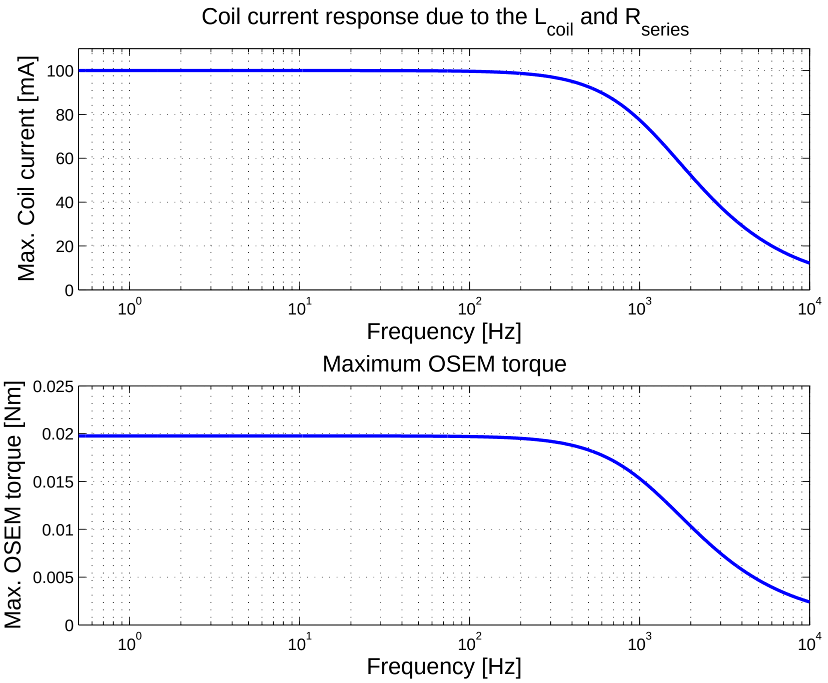

Figure 9: Coil driver output current response (top) and maximum OSEM torque response (bottom) due to the coil driver series resistor (100 Ω) and the coil inductance (13.1 mH).

[3] Kimio Tsubono, Akito Araya, Keita Kawabe, Shigenori Moriwaki, and Norikatsu Mio, *Triple-pendulum vibration isolation system for a laser interferometer*, Review of Scientific Instruments **64** (1993), no. 8, 2237–2240.

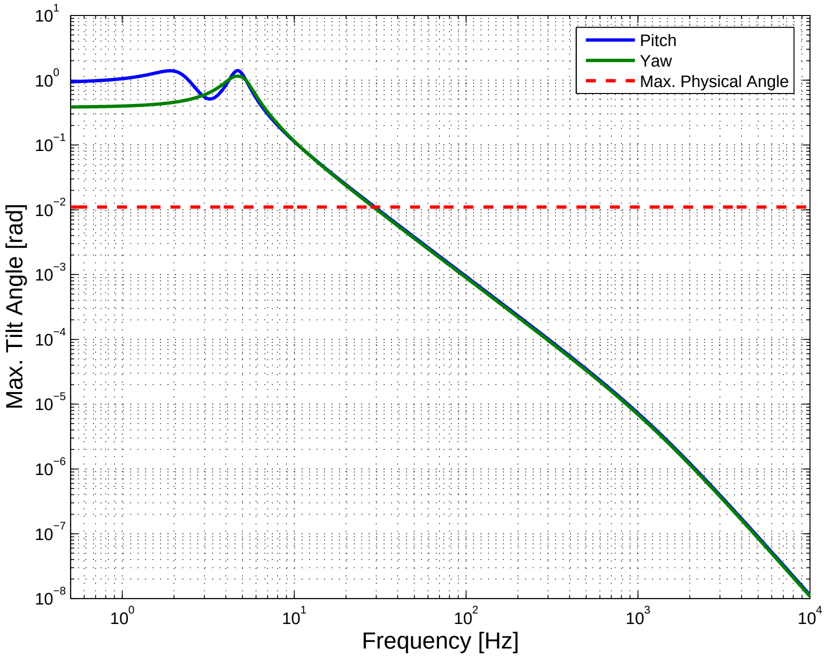

Figure 10: Max. angular deflection with maximum allowable OSEM drive current.