LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

— LIGO —

CALIFORNIA INSTITUTE OF TECHNOLOGY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Educational Activity **LIGO-T080199-v1-G** **22 August 2008**

Tautochrone:
Quantitative Exhibit Activity

Amber L. Stuver

Distribution of this document:
LIGO Science Education Center,
LIGO Scientific Collaboration

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW22-295
185 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189

http://www.ligo.caltech.edu/

Name: ____

Tautochrone:
Quantitative Exhibit Activity

Materials: Measuring Tape, Stopwatch & Calculator.

- ✓ String and a meter stick can be used in place of the measuring tape.

Before taking any measurements, interact with this exhibit. With the metal release gate resting on the top of the tracks, put a golf ball behind the gate at the top of one of the curved tracks and at the top of the straight track. Predict which ball will win the race. What physical reasons are behind your prediction?

My predicted winner: *Straight track* *Curved track*

Reasons:

Using the tape measure, determine the length of each track. Be careful to make sure that the tape lies flat on each track (special attention should be paid to measuring the length of the curved track). Express your measurements in meters.

Length of *straight track*: __________________m

Length of *curved track*: __________________m

Intuition suggests that the less distance a ball has to travel, the more likely a ball is to win the race. Based on the lengths of the track, was the track length the determining factor in which ball won the race?

Yes *No*

It is also intuitive that the ball that is moving faster at the end of the race will be the winner. Use energy conservation concepts to calculate the speed of each ball at the end of the race.

Potential and Kinetic Energy:

- *Potential energy* (**U**) is the energy stored in a mass when elevated above a reference height.
- *Kinetic energy* (**K**) is the result of converting potential energy into the speed (**v**) of the mass.

<u>Conservation of energy</u> states that the kinetic energy at the end of the race will be equal to the potential energy at the beginning of the race:

$$U = K$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

Where **m** is the mass of the balls (note that *the mass cancels out and therefore doesn't matter*), **g** is the acceleration of gravity (**g** = 9.8 m/s^2), and **h** is the height of the starting point of the balls above the ending point of the track.

Measure the starting height of both tracks and the ending heights:

Starting height: ____________________m

Ending height: ____________________m

Net height (starting-ending): ____________________m

Since there isn't a difference in the starting or ending heights of either track, then we expect that the speed of each ball when it ends the race will be the same!

Calculate the speed of the balls when they end the race (make sure to use the net height):

v = ___________________m

So, it is obvious that it isn't the ending speed that determined the outcome of the race. What really matters must then be how and when each ball came to that speed. To begin to investigate this, we will calculate the average speed ($\bar{\mathbf{v}}$) of the balls on each track.

$$\bar{v} = \frac{l}{t}$$

We already have the length measurement of each track. Now, we will need to time how long it takes the ball to follow each track. Since the tracks give a close race, we will take several timings on each track and average them together:

Track:	**Straight**	**Curved**
Trial 1:	sec	sec
Trial 2:	sec	sec
Trial 3:	sec	sec
Average time:	**sec**	**sec**

Using the average time for each track, calculate the average speed of a ball on each track (make sure to use the right length and time for each track):

Straight track:

$\bar{\text{v}}$ = ___________________m/s

Curved track:

$\bar{\text{v}}$ = ___________________m/s

We now see that the track with the highest average speed is the track that wins the race. This implies that, while the balls end the race with the same speed, they do not come into that speed in the same way.

The average rate of change of speed is measured by the average acceleration:

$$\bar{a} = \frac{v_{end} - v_{start}}{t}$$

Where $\mathbf{v_{end}}$ is the ending speed, and $\mathbf{v_{start}}$ is the starting speed (which is zero since the balls start from rest). Calculate the average acceleration for each track:

Straight track:

$\bar{a}$ = ___________________ m/s^2

Curved track:

$\bar{a}$ = ___________________ m/s^2

Again, we see that the ball with the greatest average acceleration is the one who wins the race.

SUMMARY & CONCLUSIONS

The ball rolling down the curved track wins the race even though the curved track is longer than the straight track and the speed of both balls is the same at the end of the race. This is because the acceleration that each ball undergoes is not the same – the curved track ball accumulates most of its speed earlier in the race while the straight track ball accumulates its speed evenly through the course. Since gravity is the force that is accelerating the balls and pulls the balls towards the center of the Earth, the part of the course that is steepest represents the greatest acceleration.

CHECK YOUR WORK

This page is not here for you to cheat, but for you to do a self-evaluation of your work AFTER you've completed this activity. After all, if your measurements are off, so will your calculations! N.B. <u>All measurements are approximate.</u>

Did the ball with the shortest track win the race?
NO

Length of *straight track*:
2.22 m

Length of *curved track*:
2.40 m

Starting height of *both tracks*:
1.39 m

Ending height of *both tracks*:
0.22 m

Net height of *both tracks*:
1.17 m

Speed of *both balls* ending the race:
4.79 m/s

Average time for ball to complete *straight track*:
1.128 s

Average time for ball to complete the *curved track*:
0.948 s

Average speed of the ball on the *straight track*:
1.97 m/s

Average speed of the ball on the *curved track*:
2.53 m/s

Average acceleration of the ball on the *straight track*:
4.25 m/s^2

Average acceleration of the ball on the *curved track*:
5.05 m/s^2