An Upper Limit on the Amplitude of Stochastic Gravitational-Wave Background of Cosmological Origin

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A stochastic background of gravitational waves is expected to arise from a superposition of a large number of unresolved gravitational-wave sources of astrophysical and cosmological origin. It is expected to carry unique signatures from the earliest epochs in the evolution of the universe, inaccessible to the standard astrophysical observations [1]. Direct measurements of the amplitude of this background therefore are of fundamental importance for understanding the evolution of the universe when it was younger than one minute. Here we report direct limits on the amplitude of the stochastic gravitational-wave background using the data from a two-year science run of the Laser Interferometer Gravitational-wave Observatory (LIGO) [2]. Our result constrains the energy density of the stochastic gravitational-wave background normalized by the critical energy density of the universe, in the frequency band around 100 Hz, to be less than 6.9×10^{-6} at 95% confidence. The data rule out models of early universe evolution with relatively large equation-of-state parameter [3], as well as cosmic (super)string models with relatively small string tension [4] that are favoured in some string theory models [5]. This search for the stochastic background improves upon the indirect limits from the Big Bang Nucleosynthesis [1, 6] and cosmic microwave background [7] at 100 Hz.

According to the general theory of relativity, gravitational waves (GWs) are produced by accelerating mass distributions with a quadrupole (or higher) moment. Moreover, in the early phases of the evolution of the universe, they can be produced by the mechanism of amplification of vacuum fluctuations. Once produced, GWs travel through space-time at the speed of light, and are essentially unaffected by the matter they encounter. As a result, GWs emitted shortly after the Big Bang (and observed today) would carry unaltered information about the physical processes that generated them. These waves are expected to be generated by a large number of unresolved sources, forming a stochastic gravitational-wave background (SGWB) that is usually described in terms of the GW spectrum:

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\mathrm{GW}}}{df}\,, \tag{1}$$

where $d\rho_{\mathrm{GW}}$ is the energy density of gravitational radiation contained in the frequency range f to $f + df$ and ρ_c is the critical energy density of the universe [8]. Many cosmological mechanisms for generation of the SGWB exist, such as the inflationary models [9, 10], pre-big-bang models [11–13], electroweak phase transition [14], and cosmic strings [4, 5, 15, 16]. There are also astrophysical mechanisms, such as due to magnetars [17] or rotating neutron stars [18].

The physical manifestation of GWs consists of stretching and compressing the spatial dimensions orthogonal to the direction of wave propagation, producing strain in an oscillating quadrupolar pattern. A Michelson interferometer with suspended mirrors [2] is well suited to measure this differential strain signal due to GWs. Over the past decade, LIGO has built three such multi-kilometer interferometers, at two locations [2]: H1 (4 km) and H2 (2 km) share the same facility at Hanford, WA, and L1 (4 km) is located in Livingston Parish, LA. LIGO, together with the 3 km interferometer Virgo [19] in Italy and GEO [20] in Germany, forms a network of GW observatories. LIGO has completed the science run S5 (between November 5, 2005 and September 30, 2007), acquiring one year of data coincident among H1, H2 and L1, at the interferometer design sensitivities (Fig. 1).

The search for the SGWB using LIGO data is performed by cross-correlating strain data from pairs of interferometers [8]. In the frequency domain, the cross-correlation between two interferometers is multiplied by a filter function $\tilde{Q}(f)$ (c.f. online supplement):

$$\tilde{Q}(f) = \mathcal{N}\, \frac{\gamma(f)\Omega_{\mathrm{GW}}(f)H_0^2}{f^3 P_1(f) P_2(f)}\,. \tag{2}$$

This filter optimizes the signal-to-noise ratio, enhancing the frequencies at which the signal of the template spectrum $\Omega_{\mathrm{GW}}(f)$ is strong, while suppressing the frequencies at which the detector noise ($P_1(f)$ and $P_2(f)$) is large. In Eq. 2, and throughout this letter, we assume the present value of the Hubble parameter $H_0 = 72$ km/s/Mpc [21], and use $\gamma(f)$ to denote the overlap reduction function [8], arising from the overlap of antenna patterns of interferometers at different locations and with different orientations. For the H1-L1 and H2-L1 pairs the sensitivity above roughly 50 Hz is attenuated due to the overlap reduction. Since most theoretical models in the LIGO frequency band are characterized by a power law spectrum, we assume a power law template GW spectrum with index α: $\Omega_{\mathrm{GW}}(f) = \Omega_\alpha (f/100\ \mathrm{Hz})^\alpha$.

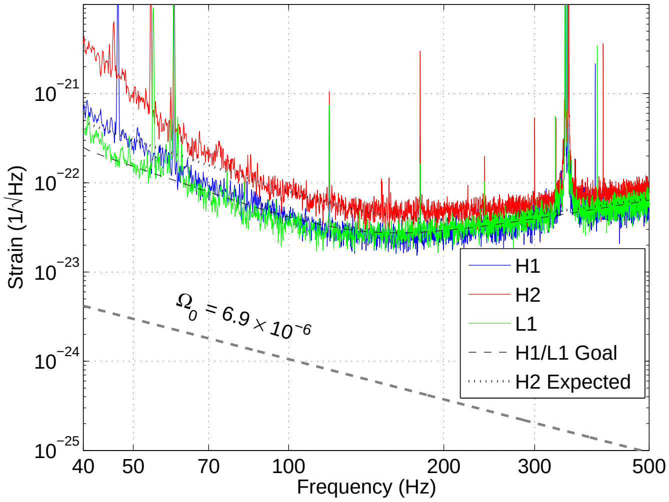

FIG. 1: **Sensitivities of LIGO interferometers.** LIGO interferometers reached their design sensitivity in November 2005, resulting in the interferometer strain noise at the level of 3×10^{-22} rms in a 100 Hz band around 100 Hz. This figure shows typical strain sensitivities of LIGO interferometers during the subsequent science run S5. Also shown is the strain amplitude corresponding to the upper limit on the GW energy density presented in this paper (gray dashed line). Note that this upper limit is $\sim$ 100 times lower than the individual interferometer sensitivities, which illustrates the advantage of using the cross-correlation technique in this analysis.

The normalization constant $\mathcal{N}$ in Eq. 2 is chosen such that the expected value of the optimally filtered cross-correlation is Ω_α.

We apply the above search technique to the data acquired by LIGO during the science run S5. We include two interferometer pairs: H1-L1 and H2-L1. Summing up the contributions to the cross-correlation in the frequency band 41.5-169.25 Hz, which contains 99% of the sensitivity, leads to the final point estimate for the frequency independent GW spectrum ($\alpha = 0$): $\Omega_0 = (2.1 \pm 2.7) \times 10^{-6}$, where the quoted error is statistical. We calculate the Bayesian 95% confidence upper limit for Ω_0, using the previous LIGO result (S4 run [22]) as a prior for Ω_0 and averaging over the interferometer calibration uncertainty. This procedure yields the 95% confidence upper limit $\Omega_0 < 6.9 \times 10^{-6}$. For other values of the power index α in the range between -3 and 3, the 95% upper limit varies between 1.9×10^{-6} and 7.1×10^{-6}. These results constitute more than an order of magnitude improvement over the previous LIGO result in this frequency region [22]. Fig. 2 shows this result in comparison with other observational constraints and some of the cosmological SGWB models.

Prior to the result described here, the most constraining bounds on the SGWB in the frequency band around 100 Hz came from the Big-Bang-Nucleosynthesis (BBN) and from cosmic microwave background (CMB) measurements. The BBN bound is derived from the fact that a large GW energy density at the time of BBN would alter the abundances of the light nuclei produced in the process. Hence, the BBN model and observations constrain the total GW energy density at the time of nucleosynthesis [1, 6]:

$$\int \Omega_{\rm GW}(f)\, d(\ln f) < 1.1 \times 10^{-5}\, (N_\nu - 3), \tag{3}$$

where N_ν (the *effective* number of neutrino species at the time of BBN) captures the uncertainty in the radiation content during BBN. Measurements of the light-element abundances, combined with the Wilkinson Microwave Anistropy Probe (WMAP) data give the upper bound $N_\nu - 3 < 1.4$ [23]. Similarly, a large GW background at the time of decoupling of CMB would alter the observed CMB and matter power spectra. Assuming homogeneous initial conditions, the total GW energy density at the time of CMB decoupling is constrained to $\int \Omega_{\rm GW}(f)\, d(\ln f) < 1.3 \times 10^{-5}$ [7]. In the LIGO frequency band and for $\alpha = 0$, these bounds become: $\Omega_0^{\rm BBN} < 1.1 \times 10^{-5}$ and $\Omega_0^{\rm CMB} < 9.5 \times 10^{-6}$. Our result has now surpassed these bounds, which is one of the major milestones that LIGO was designed to achieve. Moreover, the BBN and CMB bounds apply only to backgrounds generated prior to the BBN and the CMB decoupling respectively, while the LIGO bound also probes the SGWB produced later (this is the case, for example, in cosmic strings models).

Our result also constrains models of the early universe evolution. While the evolution of the universe following the BBN is well understood, there is little observational data probing the evolution prior to BBN, when the universe was

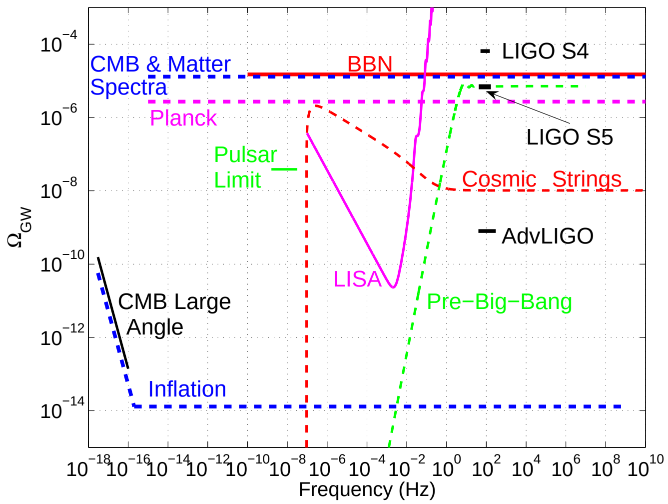

FIG. 2: **Comparison of different SGWB measurements and models.** The result presented here (LIGO S5) applies in the frequency band around 100 Hz, and is compared to the previous LIGO S4 result [22] and to the projected Advanced LIGO sensitivity [24] in this band. The indirect bounds due to BBN [1, 6] and CMB and matter power spectra [7] apply to the integral of $\Omega_{\mathrm{GW}}(f)$ over the frequency bands denoted by the corresponding dashed curves. Projected sensitivities of the satellite-based Planck CMB experiment [7] and LISA GW detector [25] are also shown. The pulsar bound [26] is based on the fluctuations in the pulse arrival times of millisecond pulsars and applies at frequencies around 10^{-8} Hz. Measurements of the CMB at large angular scales constrain the possible redshift of CMB photons due to the SGWB, and therefore limit the amplitude of the SGWB at largest wavelengths (smallest frequencies) [6]. Examples of inflationary [9, 10], cosmic strings [4, 5, 15, 16], and pre-big-bang [11–13] models are also shown (the amplitude and the spectral shape in these models can vary significantly as a function of model parameters).

less than one minute old. The GW spectrum $\Omega_{\mathrm{GW}}(f)$ carries information about exactly this epoch in the evolution. In particular, measuring $\Omega_{\mathrm{GW}}(f)$ is the best way to test for existence of presently unknown "stiff" energy components in the early universe [3], for which a small density variation is associated with a large pressure change, which could carry information about the physics of the inflationary era [27]. Fig. 3 demonstrates how the result presented here can be used to constrain the existence of these new energy components.

Our result also constrains models of cosmic (super)strings. Cosmic strings were originally proposed as topological defects formed during phase transitions in the early universe [15]. More recently, it was realized that fundamental strings may also be expanded to cosmological scales [5]. Hence, searching for cosmic strings may provide a unique and powerful window into string theory and into particle physics at the highest energy scales. Fig. 4 shows that our result, along with other observations, can be used to constrain the parameters in the cosmic string models. While our result is currently excluding a fraction of the allowed parameter space, Advanced LIGO [24] is expected to probe most of these models.

Measurements of the SGWB also offer the possibility of probing alternative models of the early universe cosmology. For example, in the pre-Big-Bang model [11–13] the universe starts off large and then undergoes a period of inflation driven by the kinetic energy of a dilaton field, after which the standard cosmology follows. Although more speculative than the standard cosmology model, the pre-Big-Bang model makes testable predictions of the GW spectrum. As shown in Fig. 5, the BBN and CMB bounds are currently the most constraining for this model and Advanced LIGO [24] is expected to surpass them.

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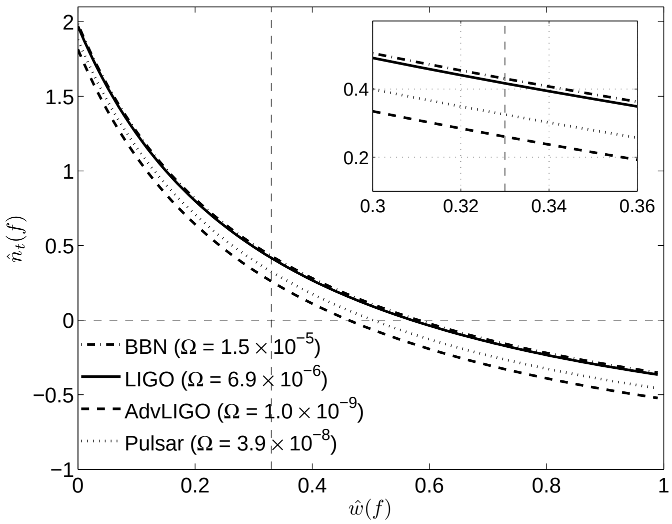

FIG. 3: **Constraining early universe evolution.** The GW spectrum $\Omega_{\rm GW}(f)$ is related to the parameters that govern the evolution of the universe [3]:
$\Omega_{\rm GW}(f) = A\ f^{\hat{\alpha}(f)}\ f^{\hat{n}_t(f)}\ r$, where $\hat{\alpha}(f) = 2\ \frac{3\hat{w}(f)-1}{3\hat{w}(f)+1}$, r is the ratio of tensor and scalar perturbation amplitudes (measured by the cosmic microwave background (CMB) experiments), $\hat{n}_t(f)$ and $\hat{w}(f)$ are effective (average) tensor tilt and equation of state parameters respectively, and A is a constant depending on various cosmological parameters. Hence, the measurements of $\Omega_{\rm GW}$ and r can be used to place constraints in the $\hat{w} - \hat{n}_t$ plane, *independently* of the cosmological model. The figure shows the $\hat{w} - \hat{n}_t$ plane for $r = 0.1$. The regions excluded by the BBN [23], LIGO, and pulsar [26] bounds are above the corresponding curves (the inset shows a zoom-in on the central part of the figure). The BBN curve was calculated in [3]. We note that the CMB bound [7] almost exactly overlaps with the BBN bound. Also shown is the expected reach of Advanced LIGO [24]. Note that these bounds apply to different frequency bands, so their direct comparison is meaningful only if $\hat{n}_t(f)$ and $\hat{w}(f)$ are frequency independent. We note that for the simplest single-field inflationary model that still agrees with the cosmological data, $V(\phi) = m^2\phi^2/2$, $r = 0.14$ and $n_t(100\text{ Hz}) = -0.035$ [28], implying a LIGO bound on the equation-of-state parameter of $\hat{w}(100\text{ Hz}) < 0.59$.

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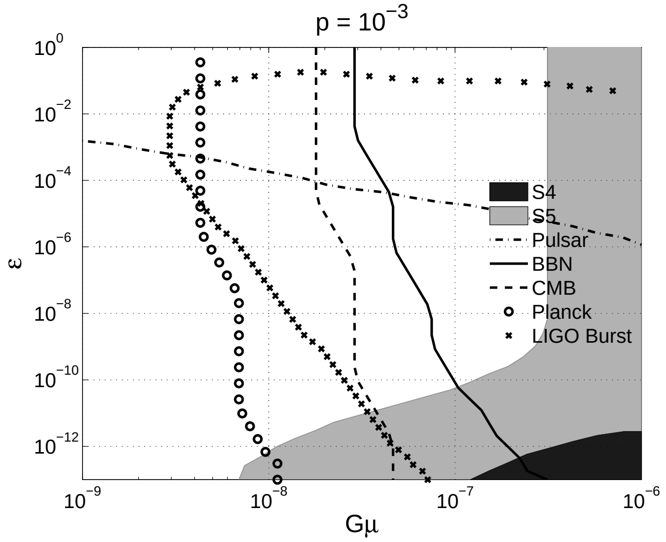

FIG. 4: **Cosmic strings models.** The network of cosmic strings is usually parametrized by the string tension μ (multiplied by the Newton constant G), and reconnection probability p. The CMB observations limit $G\mu < 10^{-6}$. If the size of the cosmic string loops is determined by the gravitational back-reaction [29], the size of the loop can be parametrized by a parameter ϵ [16] which is essentially unconstrained. The mechanism for production of GWs relies on cosmic string cusps: regions of string that move at speeds close to the speed of light. If the cusp motion points toward Earth, a detectable burst of gravitational radiation may be produced [16, 30]. The superposition of GWs from all string cusps in the cosmic string network would produce a SGWB [4]. This figure shows how different experiments probe the $\epsilon - G\mu$ plane for a typical value of $p = 10^{-3}$ [4] (p is expected to be in the range $10^{-4} - 1$). The excluded regions (always to the right of the corresponding curves) correspond to the S4 LIGO result [22], this result, BBN bound [6, 23], CMB bound [7], and the pulsar limit [26]. In particular, the bound presented in this paper excludes a new region in this plane ($7 \times 10^{-9} < G\mu < 1.5 \times 10^{-7}$ and $\epsilon < 8 \times 10^{-11}$), which is not accessible to any of the other measurements. Also shown is the expected sensitivity for the search for individual bursts from cosmic string cusps with LIGO S5 data [30]. The region to the right of this curve is expected to produce at least one cosmic string burst event detectable by LIGO during the S5 run. Note that this search is complementary to the search for the SGWB as it probes a different part of the parameter space. Also shown is the region that will be probed by the Planck satellite measurements of the CMB [7]. The entire plane shown here will be accessible to Advanced LIGO [24] SGWB search.

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Supplementary Information is linked to the online version of the paper at www.nature.com/nature.

Acknowledgements: The authors gratefully acknowledge the support of the United States National Science Foundation for the construction and operation of the LIGO Laboratory, the Science and Technology Facilities Council of the United Kingdom, the Max-Planck-Society, and the State of Niedersachsen/Germany for support of the construction and operation of the GEO600 detector, and the Italian Istituto Nazionale di Fisica Nucleare and the French Centre National de la Recherche Scientifique for the construction and operation of the Virgo detector. The authors also gratefully acknowledge the support of the research by these agencies and by the Australian Research Council, the Council of Scientific and Industrial Research of India, the Istituto Nazionale di Fisica Nucleare of Italy, the Spanish Ministerio de Educacion y Ciencia, the Conselleria d'Economia Hisenda i Innovacio of the Govern de les Illes Balears, the Royal Society, the Scottish Funding Council, the Scottish Universities Physics Alliance, The National Aeronautics and Space Administration, the Carnegie Trust, the Leverhulme Trust, the David and Lucile Packard Foundation, the Research Corporation, and the Alfred P. Sloan Foundation. This document has been assigned LIGO Laboratory

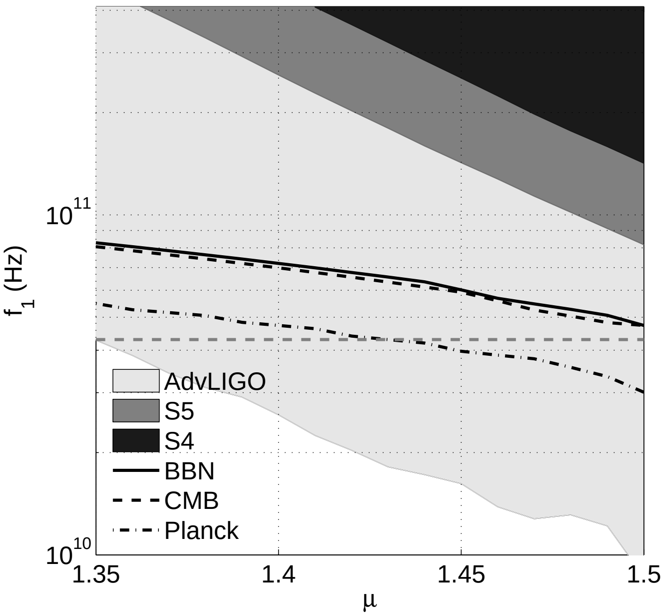

FIG. 5: **Pre-Big-Bang models.** In the pre-Big-Bang model, the GWs are produced via the mechanism of amplification of vacuum fluctuations, analogously to the standard inflationary model. The typical GW spectrum increases as f^3 up to a turn-over frequency f_s, above which $\Omega_{\mathrm{GW}}(f) \sim f^{3-2\mu}$ with $\mu < 1.5$. The spectrum cuts off at a frequency f_1, which is theoretically expected to be within a factor of 10 from 4.3×10^{10} Hz (dashed horizontal line). This figure shows the $f_1 - \mu$ plane for a representative value of $f_s = 30$ Hz. Excluded regions corresponding to the S4 result and to the result presented here are shaded. The regions excluded by the BBN [6, 23] and the CMB [7] bounds are above the corresponding curves. The expected reaches of the Advanced LIGO [24] and of the Planck satellite [7] are also shown.

document number LIGO-P080099-12-Z.

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