Gravitational Waves with Amplitude Corrections and Higher Harmonics

from

Compact Binary Coalescences.

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LIGO-G0900318-v3

What are Compact Binary Coalescences?

Briefly…

- Take two compact stellar objects i.e.,
  Black Holes and/or Neutron Stars.

- Place them in a binary orbit with each other.

- As the system evolves it will emit gravitational radiation in accordance with General Relativity.

- The gravitational radiation will take energy away from the system, reducing the orbital separation and period until eventually the objects merge.

Prove it!

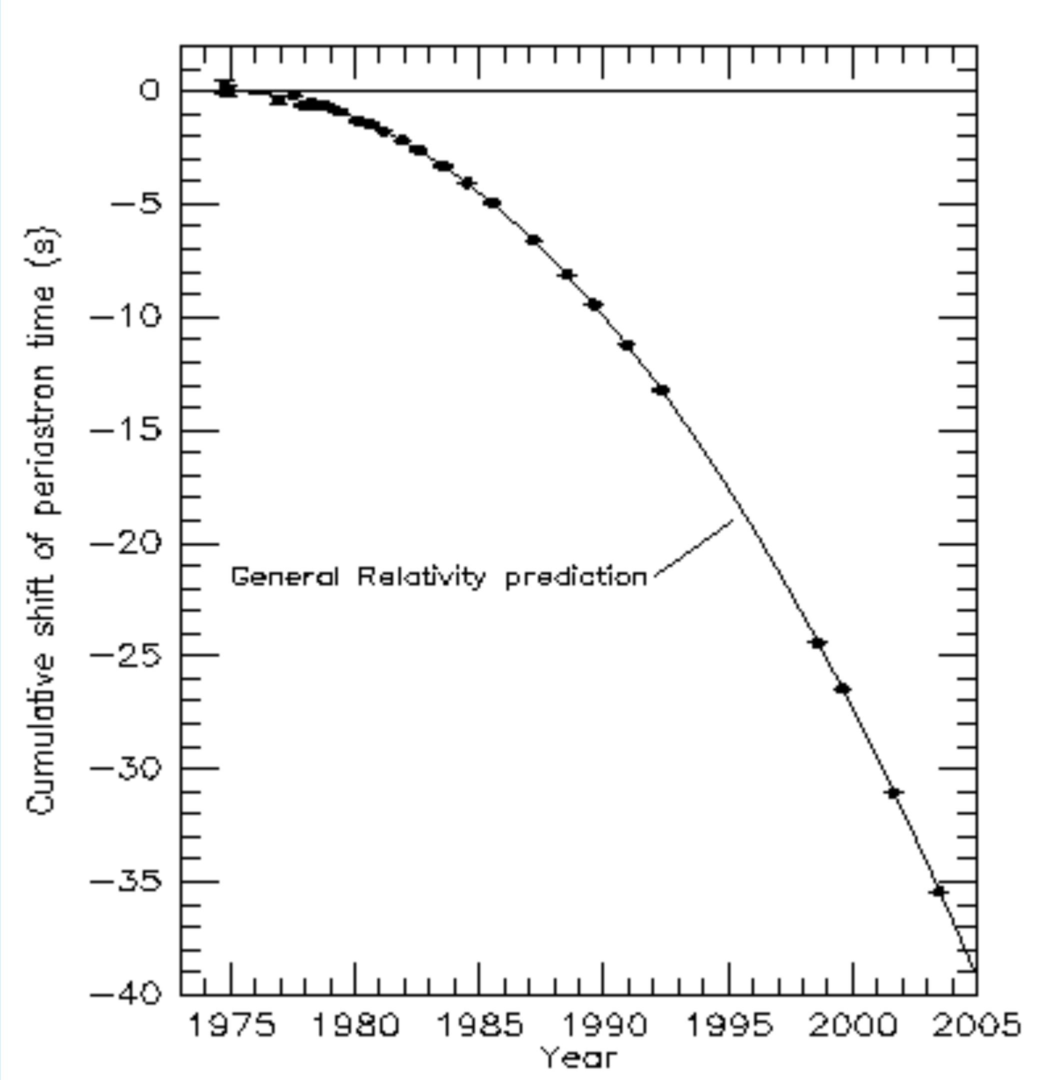
The Hulse-Taylor Binary Pulsar…

- A Binary system where at least one object is a Pulsar.

- The Pulsar gives very accurate timings of the orbital period.

- The observed decrease in the period over the last 30 years is in exact agreement with the theory of GR and gravitational wave emission.

- Many other systems have been observed with the same agreements.



Weisburg & Taylor (2004)

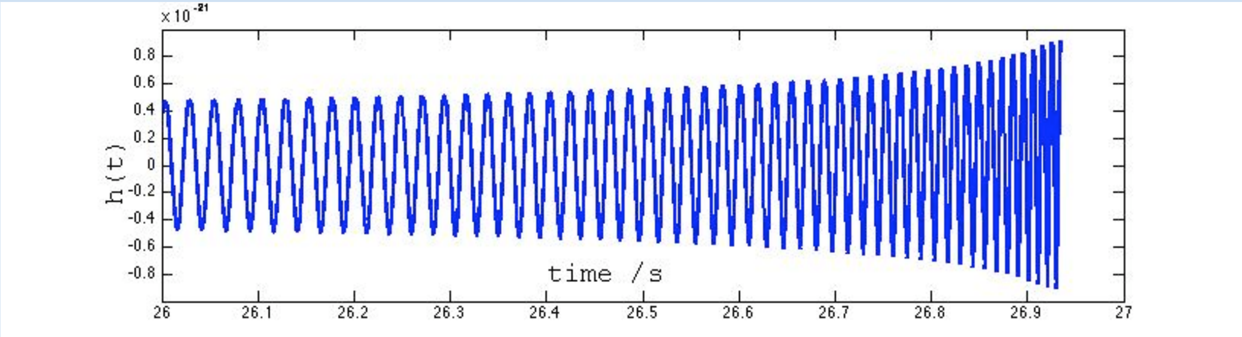
What do the gravitational waves look like?

Inspiral stage is accurately modeled by Post-Newtonian Theory…

• Einstein's Equations are solved perturbatively.

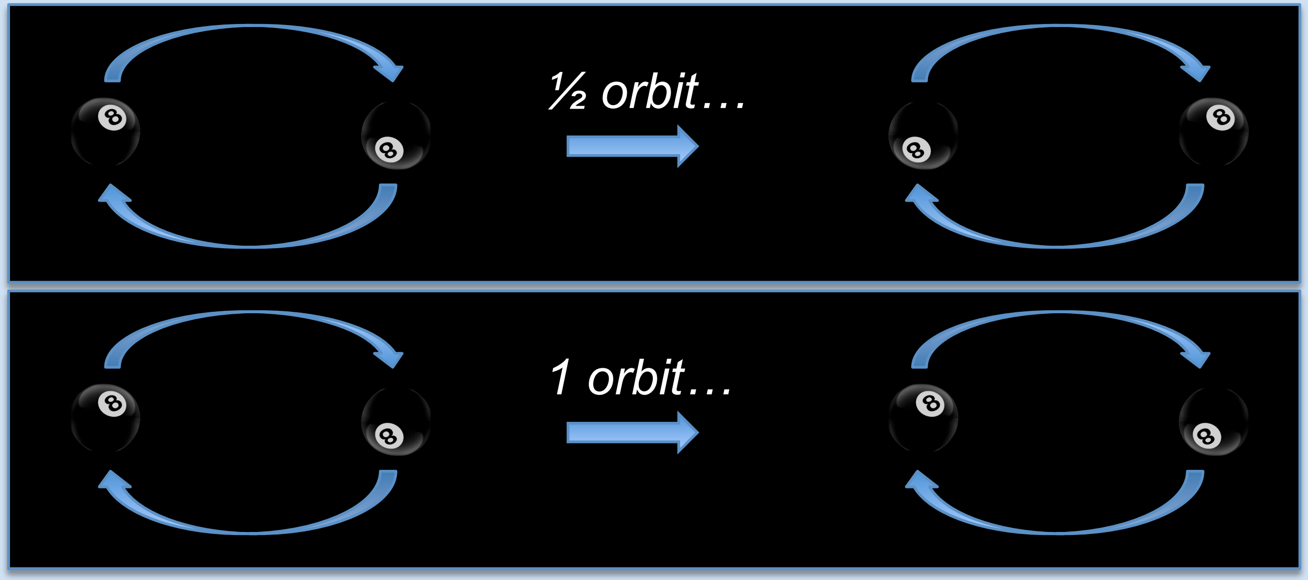
• Phases and amplitudes of gravitational waveforms are obtained as series expansions in *v/c*. This is called the Post-Newtonian Approximation.

• *Often* only the leading order term for amplitude is retained, while the phasing is taken to high order in *v/c*. This is the 'restricted' waveform as opposed to the 'full' waveform, which also retains the higher order contributions to the amplitude.



Gravitational Waves &Harmonics

- One half orbit returns the system to its start, so GW frequency is twice the orbital frequency.



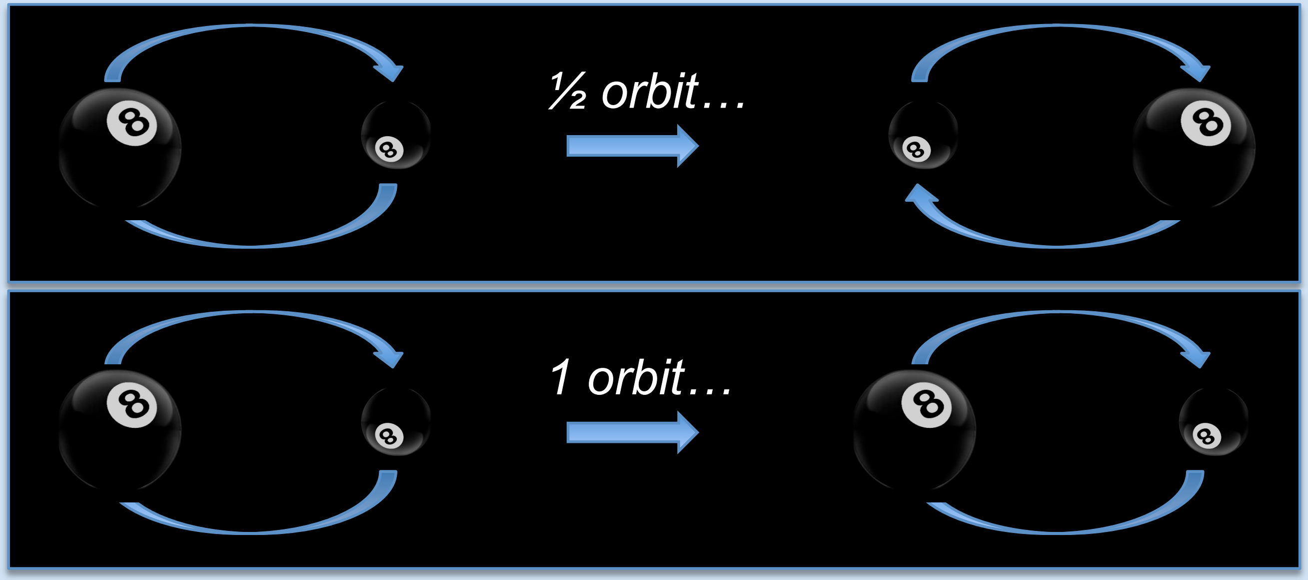
- *So the 2nd harmonic is the dominant harmonic. This is the only harmonic in the restricted waveform.*

- The full waveform contains other harmonics of the orbital frequency.

Gravitational Waves & Harmonics

- If the masses are equal then there only even harmonics of the orbital frequency.

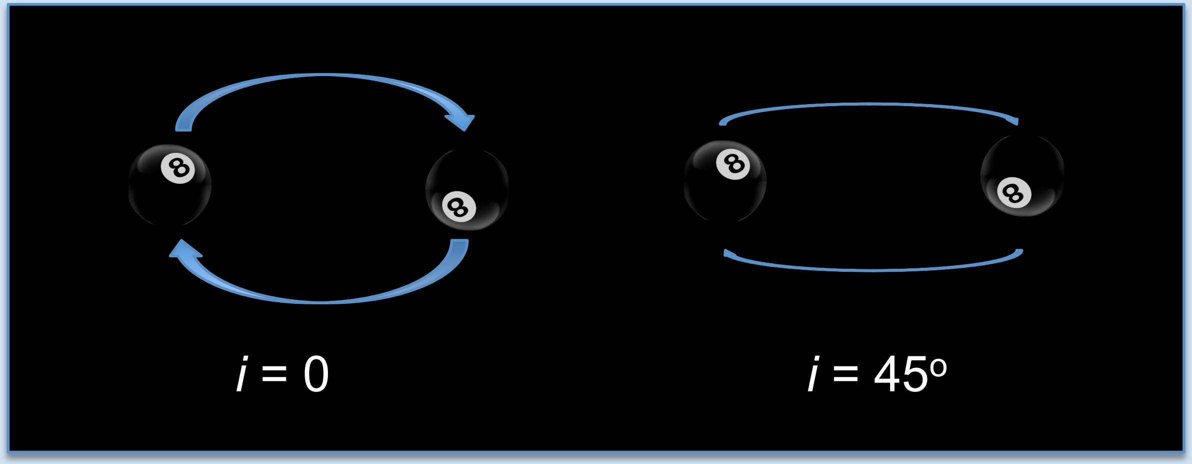
- In the case of unequal masses, odd harmonics are present, the most prominent being the 1st and 3rd harmonics.



- There is a greater difference between the full and restricted waveforms for unequal mass systems.

Gravitational Waves & Harmonics

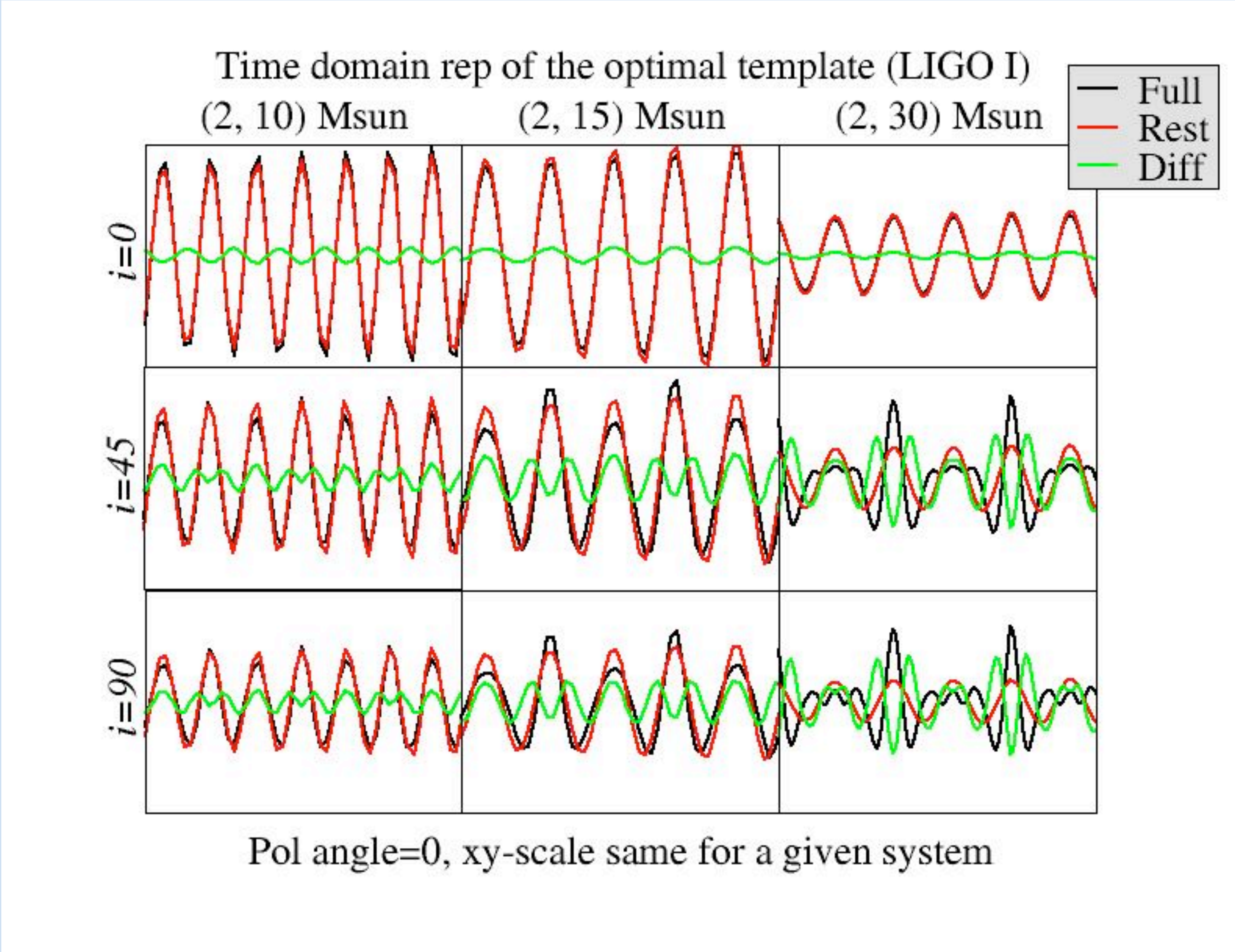
The inclination angle…



- When $i \neq 0$ there is motion towards and away from the observer and the waveform amplitude is modulated.
- This leads to further differences between the full and restricted waveforms.

Full & Restricted Waveforms

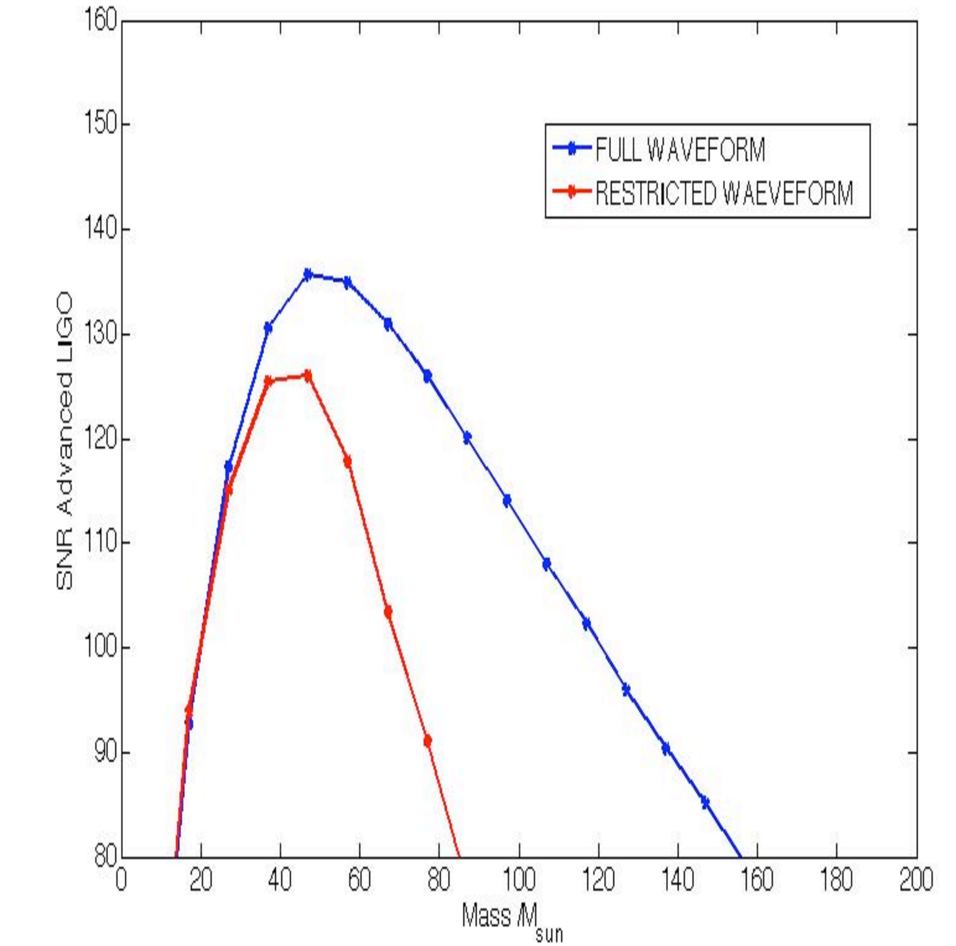
Differences…



What can we do with them?

Extend Mass Reach…

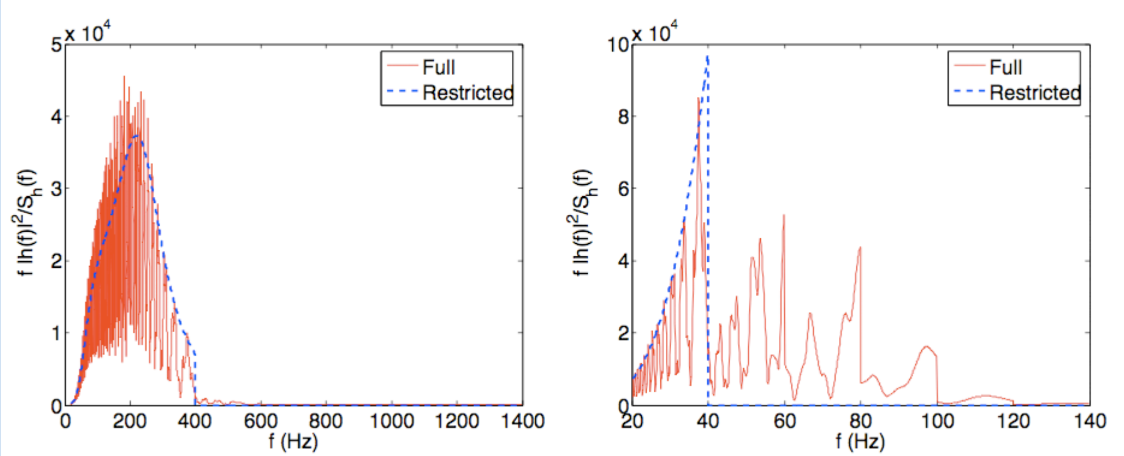
- Heavier systems reach the merger emitting GWs at lower frequencies than lower mass systems.
- All IFO detectors are limited by some lower cut off frequency (LIGO ~ 40Hz)
- Higher harmonics have power at higher frequencies, so including higher harmonic in the signals extends the mass reach of the detector.
- For example a higher harmonic may end above 40Hz, but not the dominant harmonic.

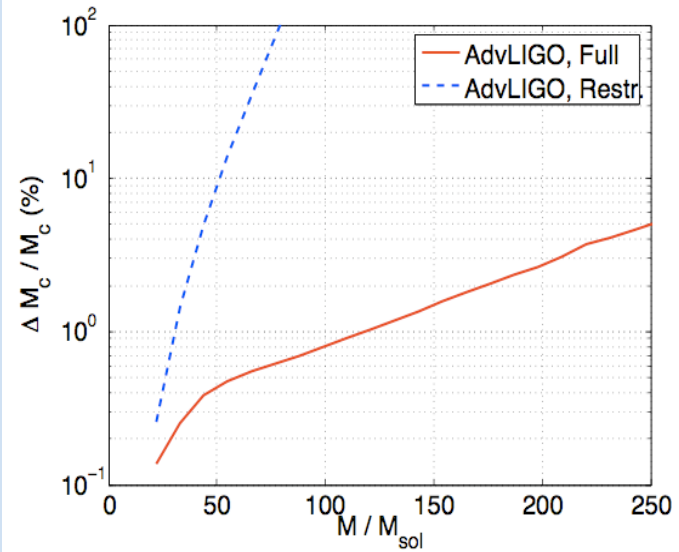


What can we do with them?

Parameter Estimation…

- The full waveform contains more information than the restricted waveform.
- In the case of the restricted waveform intrinsic information about a source can only be obtained from the phasing.
- Taking into account the amplitude corrections present in the full waveform can dramatically reduce the uncertainties in parameter estimation.



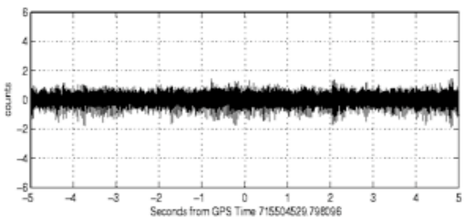


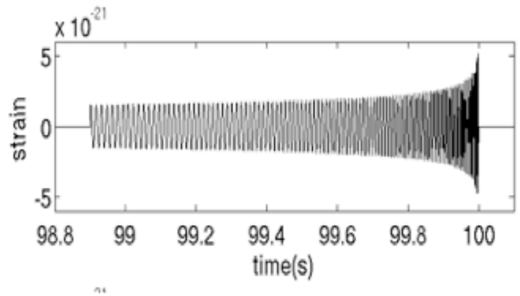
Van Den Broeck & Sengupta

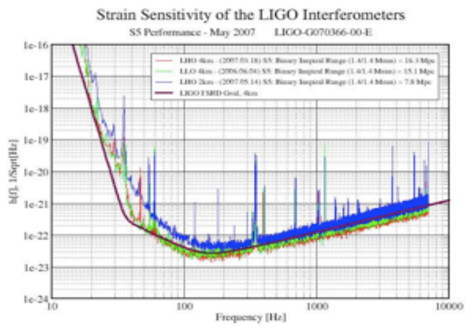
How do we find signals in data?

Matched-Filtering…

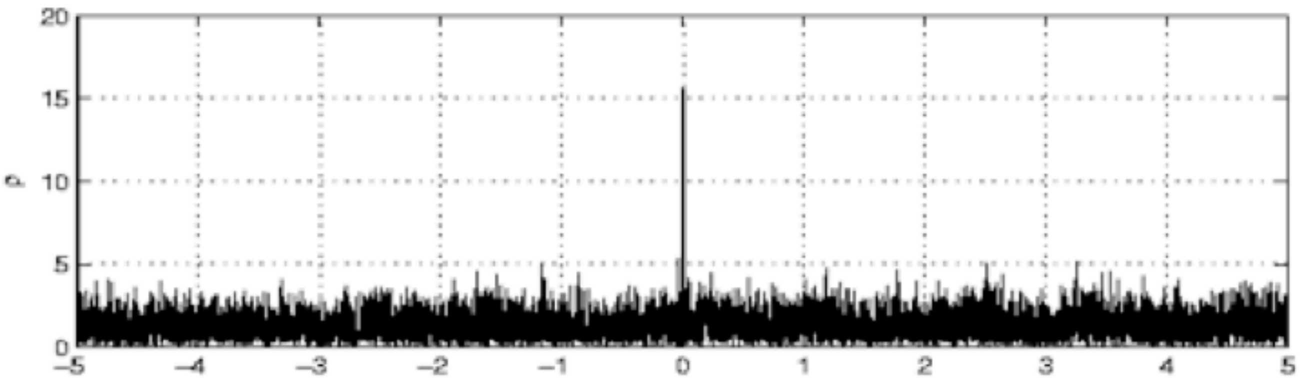
DATA X TEMPLATE / SENSITIVITY







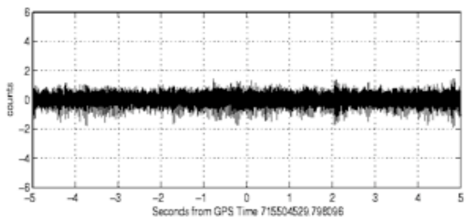
= Signal to Noise Ratio

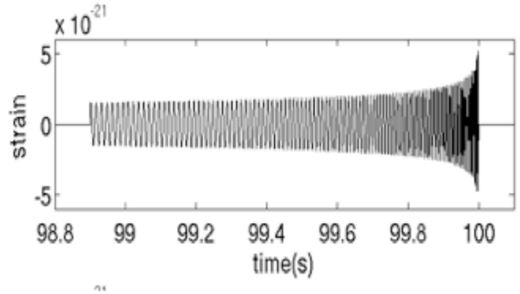


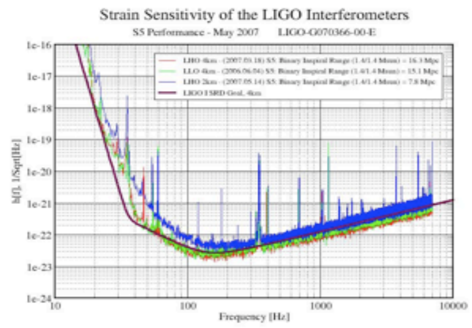
How do we find signals in data?

Matched-Filtering…

DATA X TEMPLATE / SENSITIVITY







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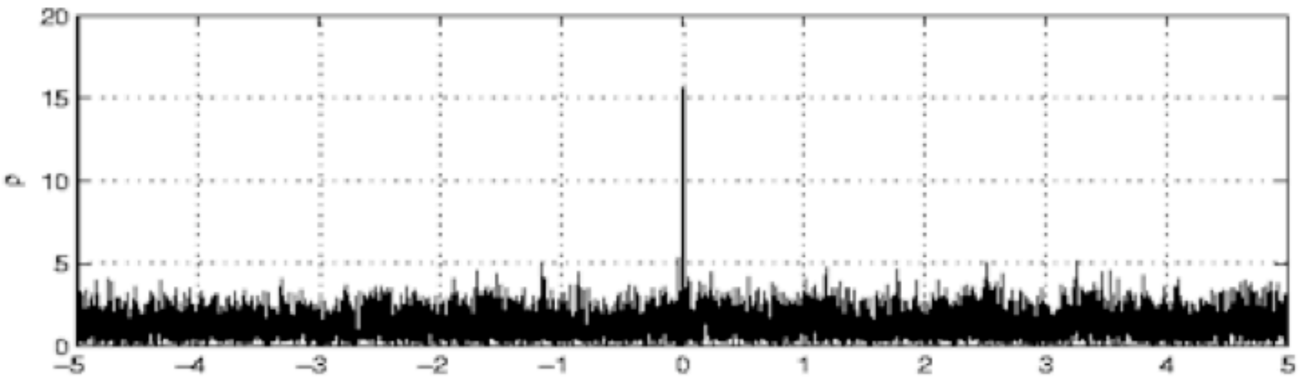


Restricted Vs. Full Templates

Restricted Template…

$$h(t) = A_0(t)\, cos[\, 2\phi(t) + \varphi_0\,]$$

- Analytically maximise the SNR over φ_0 .
- Easy to normalise such that $<h,h> = 1$.

Amplitude corrected template at only one order beyond leading order...

$$h(t) = A_1(t)\, cos[\, \phi(t) + \varphi_1\,] + A_0(t)\, cos[\, 2\phi(t) + \varphi_0\,] + A_3(t)\, cos[\, 3\phi(t) + \varphi_3\,]$$

- Cannot easily analytically maximise SNR over $\varphi_{0,1,\,3}$.
- What about normalisation?

Dealing with the Full Template

Amplitude corrected template…

$$h(t) = A_1(t)\cos[\,\phi(t) + \varphi_1\,] + A_0(t)\cos[\,2\phi(t) + \varphi_0\,] + A_3(t)\cos[\,3\phi(t) + \varphi_3\,]$$

$$h(t) = h_1(t) + h_0(t) + h_3(t)$$

- But don't we just have 3 filters? Why not filter each separately and compute the sum of squares of the SNRs?

- Yes, provided there is no correlation between the harmonics, i.e., the cross terms should be equal to zero.

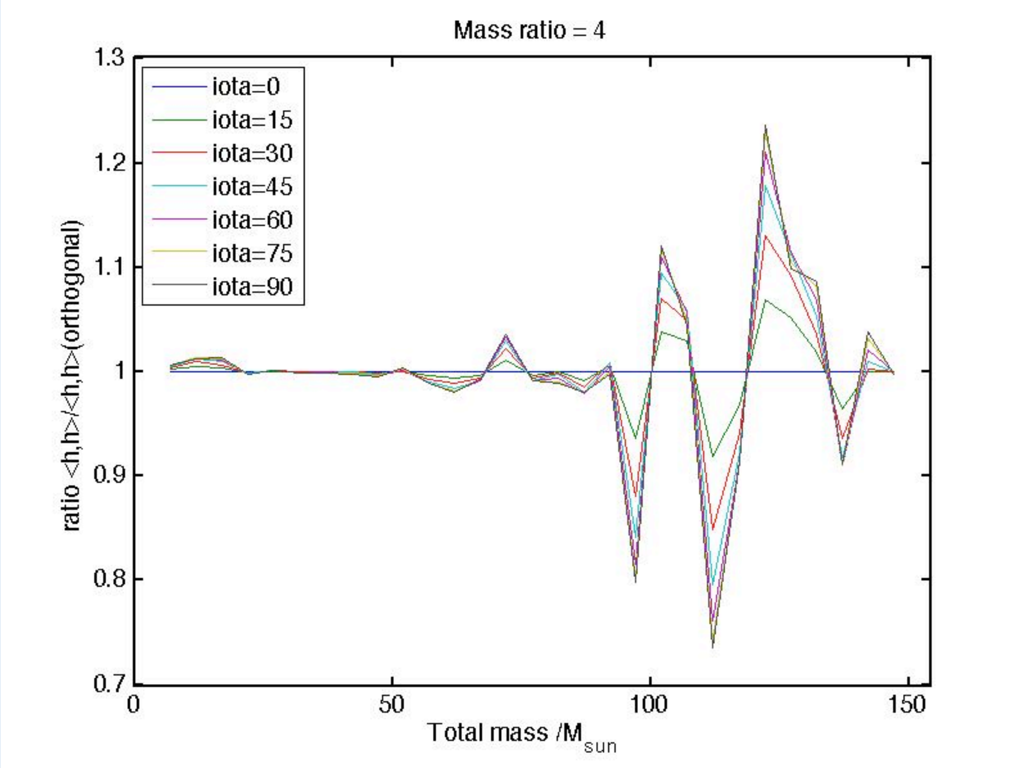
$$\langle h, h\rangle = \langle h_1, h_1\rangle + \langle h_0, h_0\rangle + \langle h_3, h_3\rangle + 2\cancel{\langle h_1, h_0\rangle} + 2\cancel{\langle h_1, h_3\rangle} + 2\cancel{\langle h_0, h_3\rangle}$$

Dealing with the Full Template

Is there correlation between the harmonics? Yes…

- If the cross-correlation is zero then the following ratio would equal 1.

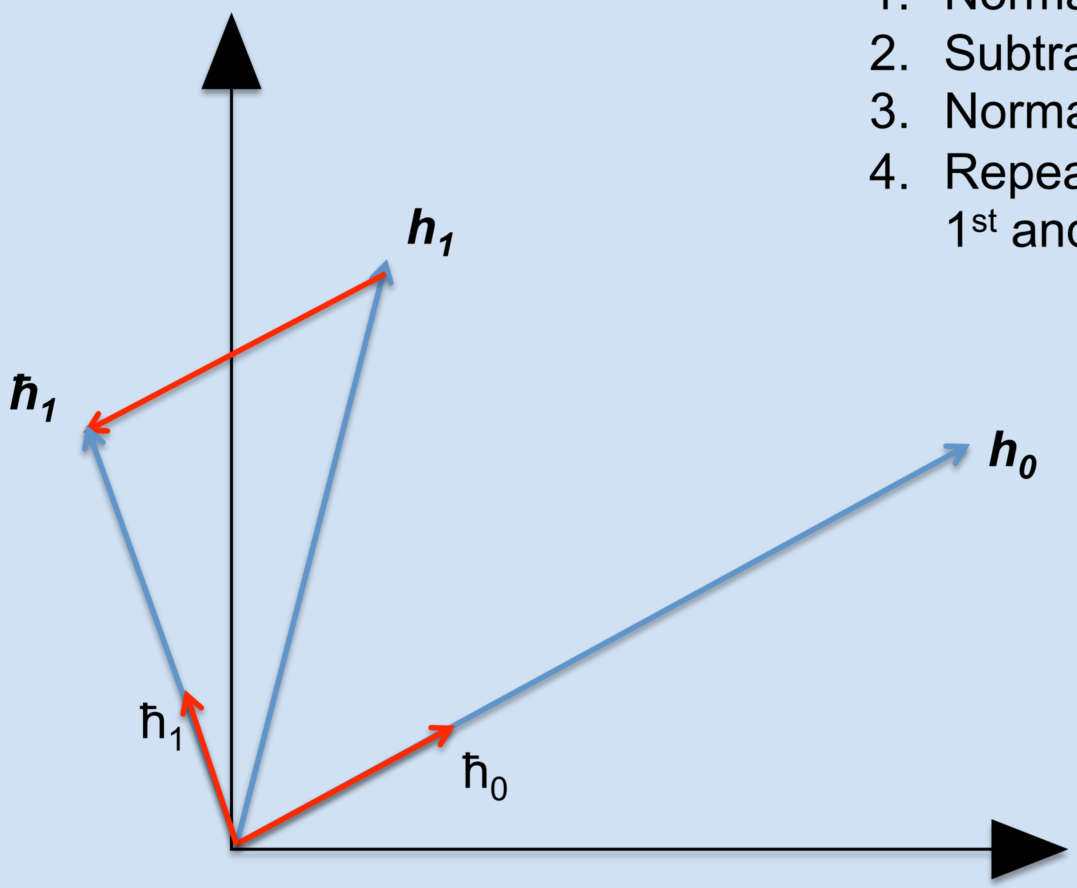
$$\text{Ratio} = \frac{\langle h_1, h_1\rangle + \langle h_0, h_0\rangle + \langle h_3, h_3\rangle + 2\,\langle h_1, h_0\rangle + 2\,\langle h_1, h_3\rangle + 2\,\langle h_0, h_3\rangle}{\langle h_1, h_1\rangle + \langle h_0, h_0\rangle + \langle h_3, h_3\rangle}$$



Gram-Schmidt Orthonormalization

Make the vectors orthonormal…

1. Normalise $\boldsymbol{h_0}$ to get $\hbar_o$.
2. Subtract <h_o, $\boldsymbol{h_1}$> from $\boldsymbol{h_1}$ to get $\boldsymbol{\hbar_1}$.
3. Normalise $\boldsymbol{\hbar_1}$ to get $\hbar_1$.
4. Repeat for 3rd harmonic with both 1st and second harmonic to get $\hbar_3$.



Maximisation of SNR?

- We are now left with three orthonormal vectors $\hbar_o$, $\hbar_1$ and $\hbar_3$.
- We can construct templates from these 3 vectors. In the frequency-domain we have…

$$h = \Sigma_{j=1..3} \, (\alpha_j + i\alpha_{j+3}) \, \hbar_j$$

- For the maximisation we have 1 constraint, $<h, h> = 1$, so:

$$\Sigma_{i=1..6} \, \alpha_i^2 = 1$$

- SNR = < DATA, NORMALISED TEMPLATE >

$$\rho = < X, h > = \Sigma_{i=1..6} \, \alpha_i < X, \hbar_i >$$

ALGEBRA

$$\rho_{MAX} = (\Sigma_{i=1..6} < X, \hbar_i >^2)^{1/2}$$

What Next?

With the orthonormalised harmonics we have constructed an amplitude corrected filter. What next?

- Investigate the use of the amplitude corrected filter in a real search for gravitational waves. What do we gain? Is it worthwhile?
- Search for simulated signals using template banks with these filters to study the parameter estimation benefits using a Monte Carlo style method.
- Can we reduce false alarm rate (by using parameter-based vetoes) in gravitational wave searches with amplitude corrected templates?
- Can we filter with *even* higher order amplitude corrected templates?
- The use of higher harmonics is expected to become more significant for advanced detectors.

Thanks for listening

Any Questions… ?