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Optical Layout and Parameters for the Advanced LIGO Cavities

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1 Introduction

1.1 Purpose and Scope

This document describes the optical parameters of the various cavities for Advanced LIGO. The lengths between various optical elements, ROC values of the mirrors, and their tolerances are listed. The various cavity parameters like Finesse, linewidths, and transversal mode spacing are calculated. Included also are the higher order modes offsets from resonances. The recycling cavity parameters are picked assuming that the TCS keeps the IFO same for both the cold as well as full power operation. The RC can be matched to a low power IFO state by appropriately changing the recycling cavity mirrors positions.

1.2 Definitions

Finesse: Measure of the selectiveness/build-up of the cavity given by $F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$

Free Spectral Range: FSR is given by FSR = $\frac{c}{2L}$ where c is the speed of light while L is the length of the cavity. The units we use are Hz.

Linewidth: The point at which the normalized transmission through a cavity becomes 1/2. This is calculated as Linewidth = $\frac{0.5 * FSR}{F}$ = Half-Width-Half-Max (HWHM).

Transversal Mode Spacing: Transversal mode spacing is the frequency difference between two Gaussian modes. For example, this is the frequency difference between TEM_{00} mode and TEM_{01}. For any higher order TEMnm mode, the difference between TEMoo and TEMnm mode is given by $\frac{(n+m)FSR * a\cos(\pm\sqrt{g})}{\pi}$ where g is the G-factor of the cavity. Note that we will use Hz as the units of transversal mode spacing.

Sagitta or Sag: For a beam with 1/e^2 beam size of w incident on a mirror of ROC R, the sag is given by $\frac{w^2}{2R}$ between the center of the beam and the beam radius.

1.3 Acronyms

ROC: Radius of Curvature

PRC: Power Recycling Cavity

SRC: Signal recycling Cavity

1.3.1 LIGO Documents

1. Michael Smith and Dennis Coyne, “Stable Recycling Cavity Mirror Coordinates and Recycling Cavity Lengths,” LIGO-T080078-06-D.
2. Muzammil A. Arain and Guido Mueller, "Design of the Advanced LIGO recycling cavities," Opt. Express 16, 10018-10032 (2008) http://www.opticsinfobase.org/abstract.cfm?URI=oe-16-14-10018.
3. R. Abbott et al., “Advanced LIGO Interferometer Sensing and Control Conceptual Design,” LIGO-T070247-00-I.
4. http://ilog.ligo-wa.caltech.edu:7285/advligo/Pickle_results?action=AttachFile&do=get&target=ASC_07May09.ppt
5. Source for as-built mirror parameters: https://nebula.ligo.caltech.edu/optics/
6. R. Abbott et al., “Advanced LIGO Length Sensing and Control Final Design, LIGO-T1000298

2 Optical Configuration

The optical configuration of the Advanced LIGO cavities is given in Fig. 1 where we include both recycling cavities and the arm cavities.

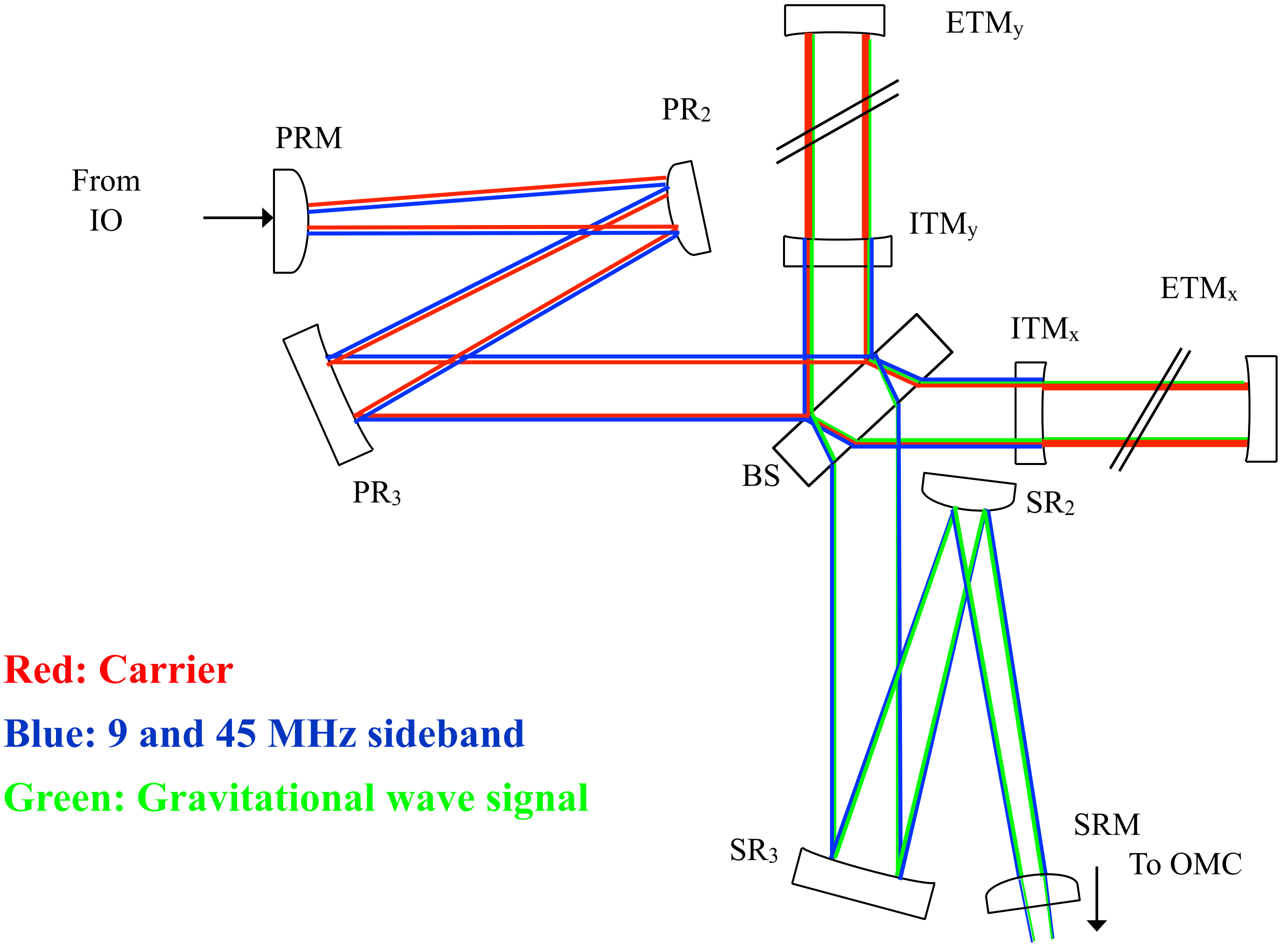

Fig. 1: Optical layout of Advanced LIGO cavities.

The optical parameters and distances are shown in Table 1. The radii of curvatures are the as-built radii from Reference 5; we used the LIGO measured data where available and vendor data for the rest. Note that the mode matching depends critically on the PR2-PR3 (SR2-SR3) distance and the radii of curvature of these two mirrors as the spatial mode between them is highly divergent. All other distances and radii of curvatures are less critical for mode matching. However the overall length of the PR (SR) cavity has to be maintained to match up with the modulation frequencies provided by ISC [6].

Table 1: **Design** Parameters and Distances in Advanced LIGO Cavities
As-Built are in below.

Recycling Cavity Parameters 25° PRC and 19° SRC Gouy Phase and 0 W Power Level

Definition	***Unit***	***PRC***		***SRC***	
		Straight	Folded	Straight	Folded
P(S)RM radius of curvature	m	-10.997	-8.8691	-5.6938	-11.3984
Distance b/w P(S)RM and P(S)R$_2$	m	16.6037	15.7971	15.726	15.941
P(S)R$_2$ ROC	m	-4.555	-4.41	-6.427	-4.894
Distance b/w P(S)R$_2$ and P(S)R$_3$	m	16.1558	15.2065	15.4607	16.0079
P(S)R$_3$ ROC	m	36	34	36	36
Distance b/w P(S)R$_3$ and BS	m	19.5384	19.4221	19.368	20.1072
BS Effective thickness	mm	0	0	131.5	132
Distance b/w BS and CP	m	4.8497$^{\#}$	9.4767$^{\#}$	4.8046$^{\Psi}$	9.4314$^{\Psi}$
Distance b/w CP and ITM	mm	5	5	5	5
ITM ROC	m	1934	1934	1934	1934
Reqd. beam waist size in arm	mm	12.0	12.01	12.0	12.01
Beam Size at ITM*	mm	53.0	53.1	53.0	53.1
Beam waist location from ITM	m	1834.2	1835	1834.2	1835
Arm Cavity Length	m	3994.5	3996.0	3994.5	3996.0
ETM ROC	m	2245	2245	2245	2245
Beam Size at ETM*	mm	62.0	62.1	62.0	62.1
Schnupp Asymmetry (X$_{SML}$-Y$_{SML}$)	mm	50.4	-50	50.4	-50
Angle of Incidence at P(S)R$_2$	degree	0.79	0.963	0.87	0.878
Angle of incidence at P(S)R$_3$	degree	0.615	1.144	0.785	0.916

SML = Short Michelson Length, * Beam Size mentioned are 1/e^2 (Intensity) beam radius, $^{\#}$ Y arm, $^{\Psi}$ X arm

Table 2: Design Component Parameters for Recycling Cavity Mirrors

Optics	ROC (m)		Beam Size (mm)		Sag (μm)		ROC Tolerance in % and mm			Tol. Sag (nm)	
	Straight	Folded	Straight	Folded	Straight	Folded	Both (%)	Straight (mm)	Folded (mm)	Straight	Folded
PRM	-11.00	-8.87	2.2	2.1	-0.23	-0.24	1	-110.0	-88.7	-2.3	-2.4
PR2	-4.56	-4.41	6.2	6.3	-4.18	-4.54	0.5	-22.8	-22.1	-20.8	-22.6
PR3	36.00	34.00	54.0	54.5	40.46	43.62	0.5	180.0	170.0	201.3	217.0
SRM	-5.69	-11.40	2.1	2.6	-0.38	-0.31	1	-56.9	-114.0	-3.8	-3.0
SR2	-6.43	-4.89	8.2	6.6	-5.27	-4.47	0.5	-32.1	-24.5	-26.2	-22.3
SR3	36.00	36.00	54.0	54.2	40.50	40.80	0.5	180.0	180.0	201.5	203.0

Note: Here ‘Sag’ is the sagitta change due to ROC while ‘Tol Sag’ is the change in sagitta between the nominal ROC value and when the ROC is at the end of the tolerance. For example, for PRM, ‘Tol. Sag’ = (Beam size)^2/(2*11) - (Beam size)^2/{2*(11+0.1)}

Note that the tolerances of P(S)R$_3$ are based upon our ability to correct any manufacturing tolerance by repositioning P(S)R$_2$. From layout standpoint, we can reposition P(S)R$_2$ by ± 10 cm requiring P(S)RM be moved by ± 20 cm. Thus we had to select 0.5% tolerance for P(S)R$_3$. Any error in ROC of P(S)R$_2$ and P(S)RM can also be corrected by repositioning the mirrors but the range of motion required for these mirrors is small.

2.1 Derived Cavity Parameters

To derive cavity parameters we have to use some mirror transmittances and distances. These are given in Table 4 and are taken from Ref. [1-3].

Table 3: Derived Cavity Parameters

Quantity	***Unit***	***Straight IFO (Folded)***
ITM Transmittance	%	1.4
PRM Transmittance	%	3.0
SRM Transmittance	%	20.0
ETM Transmittance	ppm	5
PRC Length	m	57.656 (60.411)
SRC Length	m	56.008 (62.137)
Input Mode Cleaner Round Trip Length	m	32.9461(34.513)
Input Mode Cleaner Finesse		520
Arm cavity length	m	3994.5 (3996)
Lower Mod. Frequency= IMC FSR	MHz	9.099471 (8.684428)
Upper Mod. Frequency	MHz	45.497355(43.42214)
Arm cavity Finesse		450
Arm cavity FSR	KHz	37.52
Arm cavity TMS	KHz	32.453
Arm cavity Linewidth	Hz	42.33
Arm cavity G-factor		0.8303
G-factor PRC		0.8214

One way Gouy Phase PRC	Degree	25	
G-factor SRC		0.8699	
One way Gouy Phase SRC	Degree	19	
Straight Interferometer		***PRC***	***SRC***
Carrier Recycling cavity Finesse		114	26[1]
Recycling cavity FSR	MHz	2.6	2.67
Recycling cavity TMS	MHz	0.3611	0.2825
Carrier Recycling cavity Linewidth	KHz	10.98	52.68

2.2 BS and Schnupp Asymmetry

Note that we have used the HR side of the BS for designing the PRC while for the SRC, the beam passing through the BS AR side is chosen. Thus for the straight cavity, PRC is designed for the Y-arm while SRC is designed for the X-arm. For the folded cavity, PRC is designed for the X-arm while SRC is designed for the Y-arm. The difference in the resulting ROC for the cavity mirrors is very small and well within the proposed tolerance.

One important thing to note that the optical thicknesses play a different role for the cavity length (or phase) locking and the mode matching. For the optical phase or cavity length calculations, when the light beam passes through a substrate, the optical phase accumulated as n*d where n is the refractive index while d is the thickness of the material. For the case of mode matching, a substrate of thickness d is modeled as n/d. So the effective thickness is reduced. This has an important significance when considering the BS thickness and the Schnupp asymmetry. When considering the PRC X-arm, this arm travels through the BS. So the ‘optical thickness’ is larger than the actual thickness of the BS by a factor of n, i.e., the refractive index. In assigning the Schnupp asymmetry, currently X-arm has a longer arm length than the Y-arm when considering the optical phases and the cavity lengths. However, when considering the beam propagation, because of the n/d effect in the BS thickness, the beam propagating through the BS sees n/d as the thickness. This difference more or less makes the mode matching same into the X-arm and the Y-arm. The Schnupp asymmetry is reversed in the case of folded IFO which preserves the advantage because now the Y-arm beam passes through the BS.

2.3 Low Power Operation

Parameters given in Section 2 are basically for the cold IFO state using as-designed values. TCS is supposed to preserve the IFO mode as the IFO is locked at higher power. This is done by using ring heaters on the test masses and by using CO_2 beam on the compensation plate.

However, the plan is to choose a mode matching based on the expected thermal lensing of 50km inside the ITM substrates for 12.5W input power and 0.5ppm coating absorption. This should give a reasonably good mode matching from 0 to 25W input power for nominal absorption values. The design should allow to operate in this range of power levels without engaging TCS. Note that we only include a spherical thermal lens in the ITM substrate and not any higher order mode distortions.

The following table describes the parameters and components for L1 using as-built mirrors for this scenario.

Recycling Cavity Parameters 22.9° PRC and 19.1° SRC Gouy Phase and 12.5 W input power for L1 using as-built components (Similar tables will be generated for H1/H2 once the components have been selected)

Definition	**Unit**	***PRC***		***SRC***	
		Straight	Comment	Straight	
ROC: PRM, SRM	m	-11.009	PRM-02	-5.6938	Design val.
Distance b/w P(S)RM and P(S)R2	m	16.6107		15.7586	
ROC: PR2, SR2	m	-4.545	PR2-02	-6.406	SR2-04
Distance b/w P(S)R2 and P(S)R3	m	16.1647		15.4435	
ROC: PR3, SR3	m	36.027	PR3-03	35.97	SR3-01
Distance b/w P(S)R3 and BS	m	19.5381		19.3661	
BS Effective thickness	mm	0		131.5	
Distance b/w BS and CP	m	4.862		4.8046$^{\Psi}$	
Distance b/w CP and ITM	mm	20		5	
ITM ROC	m	1934	Design val.	1934	
Reqd. beam waist size in arm	mm	12.0		12.0	
Beam Size at ITM*	mm	53.0		53.0	
Beam waist location from ITM	m	1834.2		1834.2	
Arm Cavity Length	m	3994.5	Design val.	3994.5	
ETM ROC	m	2245	Design val.	2245	
Beam Size at ETM*	mm	62.0		62.0	
Schnupp Asymmetry (X_{SML}-Y_{SML})	mm	50.4		50.4	
Angle of Incidence at P(S)R2	degree	0.79		0.87	
Angle of incidence at P(S)R3	degree	0.615		0.785	

SML = Short Michelson Length, * Beam Size mentioned are 1/e^2 (Intensity) beam radius, # Y arm, $^{\Psi}$ X arm

Changing the power will change the recycling cavity modes and the mode matching between IMC and the power recycling cavity as well as between the signal recycling cavity and the OMC (not discussed here). The mode matching between the arm cavity mode and the recycling cavity, and mode matching product of AC mode, RC mode, and IMC mode is presented in Fig. 3. The mode matching between the recycling cavities and the arm cavities is fairly insensitive to the thermal lens inside the ITM; the mode adjusts itself to automatically produce the correct radius of curvature on the ITM surface and the large divergence angle between PR2 and PR3 ensures that the beam size is also fairly insensitive to this lens fixing essentially the beam parameters at the ITM HR surface.

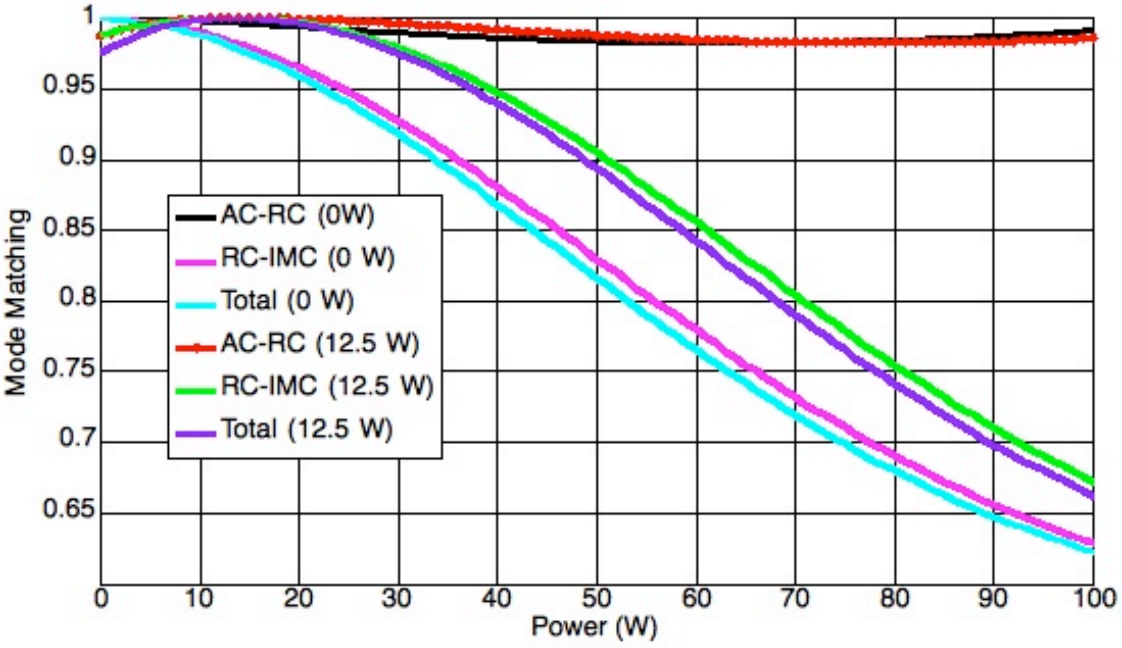

Fig. 3: Coupling between various modes for the carrier. Here RC-AC represents the coupling between the arm cavity mode and the recycling cavity mode. Total represents the product of coupling between AC-RC and IMC-RC.

The main concern here is that the modal parameters on PRM (SRM) are changing. This reduces the mode matching between IMC and the PRC (and also between SRC and OMC). Still, up to 25 W input power, the mode matching into the PRC is above about 98% and only drops at 40W to below 95%. A similar behavior is expected for the signal recycling cavity.

2.4 Changing P(S)R$_2$ Position for Optimizing for Low Power

Figure 4 shows how the mode matching changes for different input power levels if at the same time the distance between PR2 and PR3 is optimized. Note that this plot uses design ROCs and not as built ROCs; the optimal values for the as-built ROCs have to be calculated separately. However, the differences are ‘in the noise’ if the ROCs are within the requirements.

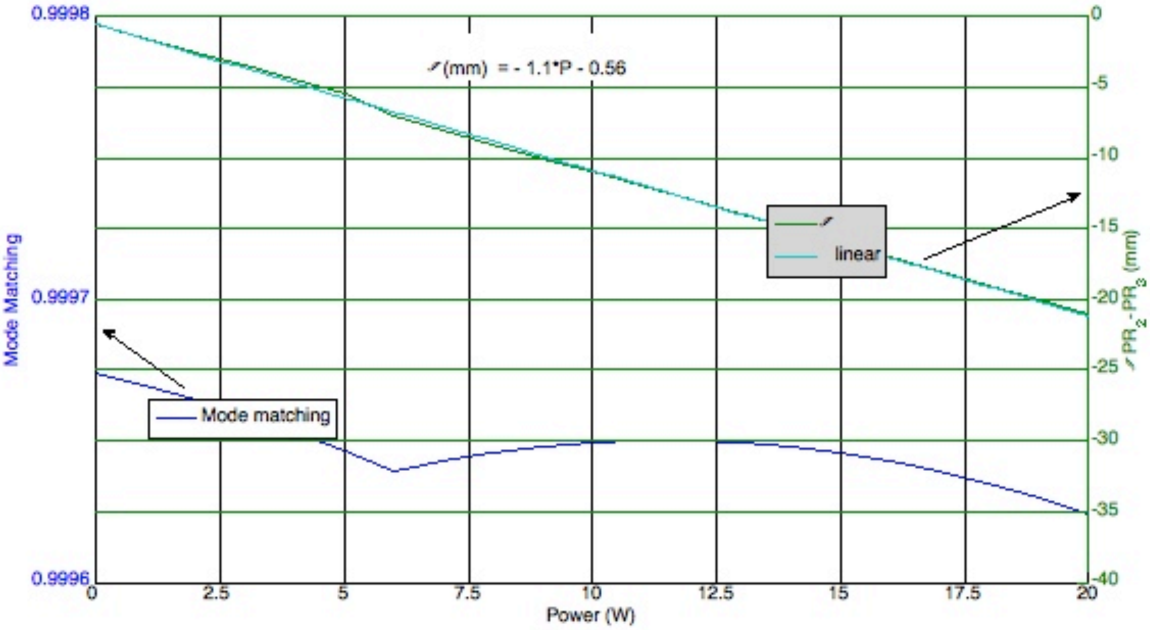

Fig. 4: This is an example of how to use the distance between PR2-PR3 to optimize the mode matching as a function of Power.

3 Summary

We have presented the optical parameters for the various cavities in Advanced LIGO. We have checked the possibility of designing the system for reduced power operation such that we do not have to engage TCS for correction. The choice of PRC and SRC Gouy phase of 25 degree and 19 degree (one way) is inspired by ISC's modeling of ASC.[4] The proposed values are not optimal for the mode matching performance but rather are a compromise between ASC and thermal performance.