Tilt tests on the RAL noise prototype – part 2

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1. MOTIVATION

In T080056 we looked at the effect that the hysteresis model would predict for the SUS. These tests were designed to check whether the SUS behaves as that model would predict.

2. EXPERIMENT SET-UP

We assembled the reaction chain of the suspension using the modified blade clamps destined for LASTI. This allowed us to check that all was well with the modified blades and clamps, as well as carry out these tests.

We fixed a small mirror to the UIM, and a second small mirror was fixed to one of the top, PU, or test masses. We made optical levers as follows. Laser pointers were arranged to shine on the mirrors and to reflect onto either a sheet of lined paper on a nearby wall, or via second mirror onto a sheet of paper on the far side of the lab. The optical lever distances, measured perpendicular to the mass faces, were 2.0m to the wall, and a total of 10.3m to the more distant target.

In the marionette test, we arranged for both reflections to fall on the near wall, at a distance of 2m from the mass suspension. We tilted the UIM by a measured amount and looked to see how far the other masses tilted. The suspension is undamped and the forced tilt is being done by hand, these results are accurate to about 5mm.

In the residual tilt tests, we arranged for the optical lever on the "output" mass to go to the far wall for higher sensitivity. With no damping on the suspension, the output spot oscillated too much to allow very precise measurements. We watched for the limits of laser spot movement, which could be done to an accuracy of about 5mm, and took an average.

3. RESULTS

3.1 Natural frequency

We checked the fundamental tilt mode of the suspension to allow a check on "d" distances. Ten oscillations took 32 seconds, giving a tilt mode of 0.31Hz.

3.2 Marrionette test

We tried input motions of +/-100mm (+/-25mRad) and +/-50mm to check for linearity.

Laser spot motion at 2m. UIM is tilted by hand, effect on other masses is observed.						
Top	75	-75	37	-37	75	-75
UIM	100	-100	50	-50	100	-100
PU	95	-90	45	-45	92	-90
Test	95	-90	45	-45	90	-90

The average motion of the other masses, per unit enforced tilt at the UIM, is

Top mass: 0.75 +/-5

PUM: 0.9 to 0.95

Test mass: 0.9 to 0.95

3.3 Residual motion tests

Before each test we disturbed the suspension by at least 50mRad and allowed it to oscillate to a stable position. One the largest oscillations had died away by air damping, we damped the remaining motion by hand to bring the residual motion at the target mass to less than ~3mRad peak-peak. It is noticeable that the results in terms of output are reasonably symmetric about this "settled" position.

test 1 - output at TM	input mm	input mRad	min	max	output mm	output mRad	%effect	Notes
oscillate			-25	25	0	0		
	100	25.00	-10	70	30	1.5	5.8%	
	-100	-25.00	-60	0	-30	-1.5	5.8%	
	100	25.00	-5	65	30	1.5	5.8%	
	-100	-25.00	-70	20	-25	-1.2	4.9%	
average output range					57.5	2.8	**5.6%**	
Output for 25mRad input					28.75	1.4		
test 2 - output at PU								
oscillate			-15	15	0	0		
	100	25.00	10	55	32.5	1.6	6.3%	
	-100	-25.00	-60	5	-27.5	-1.3	5.3%	(1)
	100	25.00	-10	45	17.5	0.8	3.4%	
	-100	-25.00	-5	-45	-25	-1.2	4.9%	
	100	25.00	15	50	32.5	1.6	6.3%	
	-100	-25.00	-10	-45	-27.5	-1.3	5.3%	
average output range					59.17	2.9	**5.7%**	
Output for 25mRad input					29.58	1.4		
test 3 - output at UI								
oscillate					0	0		
	100	25.00	10	55	32.5	1.6	6.3%	
	-100	-25.00	-5	-50	-27.5	-1.3	5.3%	
	100	25.00	20	45	32.5	1.6	6.3%	
	-100	-25.00	-10	-50	-30	-1.5	5.8%	
average output range					61.25	3.0	**5.9%**	
Output for 25mRad input					30.63	1.5		
test 4 - output at TM								
oscillate					0	0		
	100	25.00	10	45	27.5	1.3	5.3%	
	-100	-25.00	0	-55	-27.5	-1.3	5.3%	
	100	25.00	0	45	22.5	1.1	4.4%	
	-100	-25.00	-10	-45	-27.5	-1.3	5.3%	
	100	25.00	10	35	22.5	1.1	4.4%	
	-100	-25.00	-5	-50	-27.5	-1.3	5.3%	
average output range					51.67	2.5	**5.0%**	
Output for 25mRad input					25.83	1.3		

(1) for this result we overshot on returning to zero input tilt. The result is ignored in calculating average range.

4. COMPARISON WITH RESULTS FROM MODEL

4.1 Fundamental pitch mode

(Appendix 2 PitchReckoner200803098.pdf). the pitch mode of the model is 0.32Hz which compares well with the measured value of 0.31 Hz and suggests that the d distances are about right.

4.2 Marrionette behaviour

The predicted and measured marionette results are

	Measured	Modelled
Top	70-80	77
UIM	100	100
PUM	90-95	81
test	90-95	79

4.3 Residual tilts

The tilts at the various masses are known, and so the torque generated by the hysteresis can be modelled at each mass as follows (after T080056)

				Wires per mass				total wires	torque per wire	torque on mass
Mass	Wires from above	Wires going down	tilt of mass	1.05mm	0.71 mm	0.63mm	0.46 mm		2.50E-05	
				A	B	C	D		Nm/mRad	Nm
Top	2*A	4*B	18.75	2	4			6	4.69E-04	2.81E-03
UIM	4*B	4*C	25		4	4		8	6.25E-04	5.00E-03
PUM	4*C	4*D	22.5			4	4	8	5.63E-04	4.50E-03
test	4*D		22.5				4	4	5.63E-04	2.25E-03

Using the data from Mark we can generate a matrix of pitch effects at masses, and from that we can assess the effect of the combination of torques above, for a 25mRad input. We can then compare that with the effects seen in 3.3 above.

Here is the matrix, with a test to show that with input values as supplied to Mark, we get the same output by linear superposition as he got by running the full model (appendix 3 JustinTiltTest20080308.pdf)

		input at					vector		effect
		Top	UIM	PU	Test		mNm		Rad
Output at	Top	0.0001153	0.0000825	0.000067	0.0000651		1.5		0.000537
	UIM	0.0000825	0.0001063	0.0000863	0.000084		2		0.000593
	PU	0.0000667	0.0000863	0.000131	0.0001274		2		0.000662
	Test	0.0000652	0.000084	0.0001274	0.000159		1		0.00068

The effects tie up well with the results form the full model.

So we can with confidence apply the same method to the torques we expect from the hysteresis model:

input at					vector		effect	Predicted	observed
Top	UIM	PU	Test		mNm			mRad	mRad
0.0001153	0.0000825	0.000067	0.0000651		2.81E+00		0.001185	1.18	1.4
0.0000825	0.0001063	0.0000863	0.000084		5.00E+00		0.001341	1.34	1.4
0.0000667	0.0000863	0.000131	0.0001274		4.50E+00		0.001495	1.50	1.5
0.0000652	0.000084	0.0001274	0.000159		2.25E+00		0.001534	1.53	1.3

Considering the number of assumptions that have gone in to the modelling, these are certainly of the right order and in fact rather close.

5. COMMENTARY

1. The natural frequency of the suspension as built is very close to that of the model suggesting no major errors in d distance.

2. The marionette behaviour is repeatable and similar to that suggested by the model – it does not appear that we have any problems with slipping clamps or blades.

3. The residual tilts we measured compare well with those predicted by the hysteresis hypothesis, given the number of assumptions that went into the predicted numbers. Note that the hysteresis hypothesis does not distinguish between hysteresis in the wire or in the clamps.

Justin's tests for blade effects - NP model 20061213TM

Note: pitch0 through pitch3 are pitch angles in radians from top mass to optic. qxil and qxiz are the x (lateral) and z (working) displacements of the tip of the UIM blade springs.

■ Justin's test 1 : torque UIM to give 10 mrad

```
preeqtermlist = {{0.0353 * pitch1, gravtype}};

overrides = overridesorig;

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.0100386,\ \text{pitch2} \to -0.0115646,\ \text{pitch3} \to -0.01135,\ \text{qxil} \to -1.36449 \times 10^{-7},\ \text{qxir} \to -1.36451 \times 10^{-7},\ \text{qxll} \to -0.0000484164,\ \text{qxlr} \to -0.0000484164,\ \text{qxul} \to 2.65471 \times 10^{-7},\ \text{qxur} \to 2.65473 \times 10^{-7},\ \text{qzil} \to 4.60849 \times 10^{-8},\ \text{qzir} \to 4.45724 \times 10^{-8},\ \text{qzll} \to 5.87162 \times 10^{-7},\ \text{qzlr} \to 5.86509 \times 10^{-7},\ \text{qzul} \to 2.67382 \times 10^{-9},\ \text{qzur} \to 2.62532 \times 10^{-9},\ \text{roll0} \to 0.,\ \text{roll1} \to 0.,\ \text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 6.27558 \times 10^{-6},\ \text{x1} \to -9.94711 \times 10^{-6},\ \text{x2} \to -0.0000477445,\ \text{x3} \to -0.0000323956,\ \text{y0} \to 1.32212 \times 10^{-10},\ \text{y1} \to 1.66767 \times 10^{-9},\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,\ \text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.693,\ \text{z2} \to -1.034,\ \text{z3} \to -1.636\}$$

■ Justin's test 2 : same torque as previous with UIM blades set laterally very stiff

```
preeqtermlist = {{0.0353 * pitch1, gravtype}};

overrides = Override[overridesorig, {kbix -> scale[100]}];

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.0100382,\ \text{pitch2} \to -0.0115635,\ \text{pitch3} \to -0.0113489,\ \text{qxil} \to -1.28427 \times 10^{-9},\ \text{qxir} \to -1.28429 \times 10^{-9},\ \text{qxll} \to -0.0000484123,\ \text{qxlr} \to -0.0000484123,\ \text{qxul} \to 2.65449 \times 10^{-7},\ \text{qxur} \to 2.65631 \times 10^{-7},\ \text{qzil} \to 4.81009 \times 10^{-8},\ \text{qzir} \to 5.26396 \times 10^{-8},\ \text{qzll} \to 5.86841 \times 10^{-7},\ \text{qzlr} \to 5.8788 \times 10^{-7},\ \text{qzul} \to -2.77735 \times 10^{-10},\ \text{qzur} \to -1.05603 \times 10^{-9},\ \text{roll0} \to 0.,\ \text{roll1} \to 0.,\ \text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 5.95126 \times 10^{-6},\ \text{x1} \to -0.0000101715,\ \text{x2} \to -0.0000480708,\ \text{x3} \to -0.0000326241,\ \text{y0} \to 8.55016 \times 10^{-10},\ \text{y1} \to -3.3349 \times 10^{-9},\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,\ \text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.688137,\ \text{z2} \to -1.02914,\ \text{z3} \to -1.63114\}$$

■ Justin's test 1 : torque UIM to give 10 mrad

```
preeqtermlist = {{0.0353 * pitch1, gravtype}};

overrides = overridesorig;

Reset[All]

Calculate[optval]
```

```
optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.0100386,\ \text{pitch2} \to -0.0115646,\ \text{pitch3} \to -0.01135,\ \text{qxil} \to -1.36449\times10^{-7},$$
$$\text{qxir} \to -1.36451\times10^{-7},\ \text{qxll} \to -0.0000484164,\ \text{qxlr} \to -0.0000484164,\ \text{qxul} \to 2.65471\times10^{-7},$$
$$\text{qxur} \to 2.65473\times10^{-7},\ \text{qzil} \to 4.60849\times10^{-8},\ \text{qzir} \to 4.45724\times10^{-8},\ \text{qzll} \to 5.87162\times10^{-7},$$
$$\text{qzlr} \to 5.86509\times10^{-7},\ \text{qzul} \to 2.67382\times10^{-9},\ \text{qzur} \to 2.62532\times10^{-9},\ \text{roll0} \to 0.,\ \text{roll1} \to 0.,$$
$$\text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 6.27558\times10^{-6},\ \text{x1} \to -9.94711\times10^{-6},\ \text{x2} \to -0.0000477445,$$
$$\text{x3} \to -0.0000323956,\ \text{y0} \to 1.32212\times10^{-10},\ \text{y1} \to 1.66767\times10^{-9},\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,$$
$$\text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.693,\ \text{z2} \to -1.034,\ \text{z3} \to -1.636\}$$

■ Justin's test 2 : same torque as previous with UIM blades set laterally very stiff

```
preeqtermlist = {{0.0353 * pitch1, gravtype}};

overrides = Override[overridesorig, {kbix -> scale[100]}];

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.0100382,\ \text{pitch2} \to -0.0115635,\ \text{pitch3} \to -0.0113489,$$
$$\text{qxil} \to -1.28427\times10^{-9},\ \text{qxir} \to -1.28429\times10^{-9},\ \text{qxll} \to -0.0000484123,\ \text{qxlr} \to -0.0000484123,$$
$$\text{qxul} \to 2.65449\times10^{-7},\ \text{qxur} \to 2.65631\times10^{-7},\ \text{qzil} \to 4.81009\times10^{-8},\ \text{qzir} \to 5.26396\times10^{-8},$$
$$\text{qzll} \to 5.86841\times10^{-7},\ \text{qzlr} \to 5.8788\times10^{-7},\ \text{qzul} \to -2.77735\times10^{-10},\ \text{qzur} \to -1.05603\times10^{-9},\ \text{roll0} \to 0.,$$
$$\text{roll1} \to 0.,\ \text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 5.95126\times10^{-6},\ \text{x1} \to -0.0000101715,\ \text{x2} \to -0.0000480708,$$
$$\text{x3} \to -0.0000326241,\ \text{y0} \to 8.55016\times10^{-10},\ \text{y1} \to -3.3349\times10^{-9},\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,$$
$$\text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.688137,\ \text{z2} \to -1.02914,\ \text{z3} \to -1.63114\}$$

■ Justin's test 3 : same torque as previous with d1 smaller and dn larger by 1 mm

```
preeqtermlist = {{0.0353 * pitch1, gravtype}};

overrides =
  overrides = Override[overridesorig, {d1 → increment[-0.001], dn → increment[0.001]}];

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.0128756,\ \text{pitch2} \to -0.01483,\ \text{pitch3} \to -0.0145547,\ \text{qxil} \to -4.33044\times10^{-7},$$
$$\text{qxir} \to -4.33043\times10^{-7},\ \text{qxll} \to -0.0000627903,\ \text{qxlr} \to -0.0000627904,\ \text{qxul} \to 5.59434\times10^{-7},$$
$$\text{qxur} \to 5.59432\times10^{-7},\ \text{qzil} \to 8.69293\times10^{-8},\ \text{qzir} \to 8.09094\times10^{-8},\ \text{qzll} \to 1.06651\times10^{-6},$$
$$\text{qzlr} \to 1.06604\times10^{-6},\ \text{qzul} \to -2.32417\times10^{-9},\ \text{qzur} \to 1.25983\times10^{-9},\ \text{roll0} \to 0.,\ \text{roll1} \to 0.,$$
$$\text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 7.48757\times10^{-6},\ \text{x1} \to -9.78978\times10^{-6},\ \text{x2} \to -0.0000707413,$$
$$\text{x3} \to -0.0000508122,\ \text{y0} \to -4.39636\times10^{-9},\ \text{y1} \to 2.19682\times10^{-9},\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,$$
$$\text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.694,\ \text{z2} \to -1.034,\ \text{z3} \to -1.636\}$$

■ Justin's test 1 ALT : torque UIM to give 100 mrad (10 times what Justin suggested)

```
preeqtermlist = {{0.303 * pitch1, gravtype}};

overrides = overridesorig;

Reset[All]

Calculate[optval]
```

```
optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.100129,\ \text{pitch2} \to -0.115193,\ \text{pitch3} \to -0.112942,$$
$$\text{qxil} \to -8.11308 \times 10^{-6},\ \text{qxir} \to -0.0000336403,\ \text{qxll} \to -0.000510194,\ \text{qxlr} \to -0.000471893,$$
$$\text{qxul} \to -3.58351 \times 10^{-6},\ \text{qxur} \to -3.96948 \times 10^{-6},\ \text{qzil} \to -0.000245605,\ \text{qzir} \to -0.000245535,$$
$$\text{qzll} \to 0.000586682,\ \text{qzlr} \to 0.000582828,\ \text{qzul} \to -6.80116 \times 10^{-6},\ \text{qzur} \to -6.82879 \times 10^{-6},$$
$$\text{roll0} \to 0.,\ \text{roll1} \to 0.,\ \text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to -0.000037085,\ \text{x1} \to -0.000255276,$$
$$\text{x2} \to -0.000704751,\ \text{x3} \to -0.000559094,\ \text{y0} \to 4.14656 \times 10^{-8},\ \text{y1} \to 3.88937 \times 10^{-9},$$
$$\text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to -0.0000616451,\ \text{yaw1} \to -0.000129241,\ \text{yaw2} \to -2.85344 \times 10^{-7},$$
$$\text{yaw3} \to 1.05785 \times 10^{-6},\ \text{z0} \to -0.416007,\ \text{z1} \to -0.693261,\ \text{z2} \to -1.03373,\ \text{z3} \to -1.63572\}$$

■ Justin's test 2 ALT : same torque as previous (10 times what Justin suggested) with UIM blades set laterally very stiff

```
preeqtermlist = {{0.303 * pitch1, gravtype}};

overrides = Override[overridesorig, {kbix -> scale[100]}];

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.102811,\ \text{pitch2} \to -0.118612,\ \text{pitch3} \to -0.116277,$$
$$\text{qxil} \to -2.90011 \times 10^{-7},\ \text{qxir} \to -5.98478 \times 10^{-7},\ \text{qxll} \to -0.000498093,$$
$$\text{qxlr} \to -0.000498354,\ \text{qxul} \to 1.07092 \times 10^{-6},\ \text{qxur} \to 1.07213 \times 10^{-6},\ \text{qzil} \to -0.000294307,$$
$$\text{qzir} \to -0.000294331,\ \text{qzll} \to 0.000684178,\ \text{qzlr} \to 0.000684205,\ \text{qzul} \to -4.44402 \times 10^{-6},$$
$$\text{qzur} \to -4.42772 \times 10^{-6},\ \text{roll0} \to 0.,\ \text{roll1} \to 0.,\ \text{roll2} \to 0.,\ \text{roll3} \to 0.,$$
$$\text{x0} \to -7.26239 \times 10^{-6},\ \text{x1} \to -0.000177927,\ \text{x2} \to -0.000642081,\ \text{x3} \to -0.000480022,$$
$$\text{y0} \to -2.31447 \times 10^{-8},\ \text{y1} \to 0.,\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,\ \text{yaw1} \to 0.,\ \text{yaw2} \to -1.394 \times 10^{-6},$$
$$\text{yaw3} \to -2.788 \times 10^{-6},\ \text{z0} \to -0.416005,\ \text{z1} \to -0.688446,\ \text{z2} \to -1.02881,\ \text{z3} \to -1.6308\}$$

■ Justin's test 3 ALT : same torque as previous (10 times what Justin suggested) with d1 smaller and dn larger by 1 mm

```
preeqtermlist = {{0.303 * pitch1, gravtype}};

overrides =
  overrides = Override[overridesorig, {d1 → increment[-0.001], dn → increment[0.001]}];

Reset[All]

Calculate[optval]
```

Note: the pendulum has tipped into the unstable region and fallen over

```
optval
```

$$\{\text{pitch0} \to 3.14015,\ \text{pitch1} \to -3.13135,\ \text{pitch2} \to -0.329134,\ \text{pitch3} \to -0.478165,$$
$$\text{qxil} \to -0.0000167885,\ \text{qxir} \to -0.0000291592,\ \text{qxll} \to -0.000198507,\ \text{qxlr} \to -0.0000279311,$$
$$\text{qxul} \to 0.0000229401,\ \text{qxur} \to -0.0000652846,\ \text{qzil} \to 0.543126,\ \text{qzir} \to 0.542718,\ \text{qzll} \to 0.295982,$$
$$\text{qzlr} \to 0.292361,\ \text{qzul} \to 0.000211021,\ \text{qzur} \to -0.00119918,\ \text{roll0} \to -0.00328662,$$
$$\text{roll1} \to -0.00132596,\ \text{roll2} \to -0.00953047,\ \text{roll3} \to -0.0100082,\ \text{x0} \to -0.000142889,$$
$$\text{x1} \to 0.000909947,\ \text{x2} \to 0.00207642,\ \text{x3} \to 0.00426694,\ \text{y0} \to 0.00106433,\ \text{y1} \to 0.00355147,$$
$$\text{y2} \to 0.00439832,\ \text{y3} \to 0.00581727,\ \text{yaw0} \to -0.0484566,\ \text{yaw1} \to -0.0711835,\ \text{yaw2} \to -0.0660571,$$
$$\text{yaw3} \to 0.028501,\ \text{z0} \to -0.422571,\ \text{z1} \to -1.23839,\ \text{z2} \to -1.86842,\ \text{z3} \to -2.47013\}$$

■ Justin's test 4a : UIM immobilized in pitch, torque at PM to give 10 mrad at optic

```
preeqtermlist = {{0.5 * 10 000 * pitch1^2, gravtype}, {0.0591 * pitch2, gravtype}};
```

```
overrides = overridesorig;

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -6.47726\times10^{-6},\ \text{pitch2} \to -0.0101939,\ \text{pitch3} \to -0.0100047,$$
$$\text{qxil} \to 1.08384\times10^{-7},\ \text{qxir} \to 1.08385\times10^{-7},\ \text{qxll} \to -3.66593\times10^{-8},\ \text{qxlr} \to -3.6661\times10^{-8},$$
$$\text{qxul} \to -3.44392\times10^{-11},\ \text{qxur} \to -3.80217\times10^{-11},\ \text{qzil} \to -1.05136\times10^{-10},\ \text{qzir} \to -1.06924\times10^{-10},$$
$$\text{qzll} \to 8.67244\times10^{-8},\ \text{qzlr} \to 8.66245\times10^{-8},\ \text{qzul} \to -1.2386\times10^{-10},\ \text{qzur} \to -2.50343\times10^{-10},$$
$$\text{roll0} \to 0.,\ \text{roll1} \to 0.,\ \text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 3.68928\times10^{-6},\ \text{x1} \to 4.86751\times10^{-6},$$
$$\text{x2} \to -0.0000134631,\ \text{x3} \to 0.,\ \text{y0} \to 3.34784\times10^{-11},\ \text{y1} \to 5.26329\times10^{-11},\ \text{y2} \to 0.,\ \text{y3} \to 0.,$$
$$\text{yaw0} \to 0.,\ \text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.693,\ \text{z2} \to -1.034,\ \text{z3} \to -1.636\}$$

■ Justin's test 4b : same torque at PM as previous, UIM now free

```
preeqtermlist = {{0.0591*pitch2, gravtype}};

overrides = overridesorig;

Reset[All]

Calculate[optval]

optval
```

$$\{\text{pitch0} \to 0.,\ \text{pitch1} \to -0.019262,\ \text{pitch2} \to -0.0323481,\ \text{pitch3} \to -0.0317478,\ \text{qxil} \to -2.03077\times10^{-9},$$
$$\text{qxir} \to -2.03077\times10^{-9},\ \text{qxll} \to -0.0000933845,\ \text{qxlr} \to -0.0000933839,\ \text{qxul} \to 6.49053\times10^{-7},$$
$$\text{qxur} \to 6.49061\times10^{-7},\ \text{qzil} \to 3.71784\times10^{-7},\ \text{qzir} \to 3.71723\times10^{-7},\ \text{qzll} \to 2.9414\times10^{-6},$$
$$\text{qzlr} \to 2.94061\times10^{-6},\ \text{qzul} \to 1.79589\times10^{-9},\ \text{qzur} \to -1.09727\times10^{-9},\ \text{roll0} \to 0.,\ \text{roll1} \to 0.,$$
$$\text{roll2} \to 0.,\ \text{roll3} \to 0.,\ \text{x0} \to 0.0000249851,\ \text{x1} \to 2.2443\times10^{-6},\ \text{x2} \to -0.0000867417,$$
$$\text{x3} \to -0.000043972,\ \text{y0} \to 3.28512\times10^{-9},\ \text{y1} \to 3.28846\times10^{-9},\ \text{y2} \to 0.,\ \text{y3} \to 0.,\ \text{yaw0} \to 0.,$$
$$\text{yaw1} \to 0.,\ \text{yaw2} \to 0.,\ \text{yaw3} \to 0.,\ \text{z0} \to -0.416,\ \text{z1} \to -0.693,\ \text{z2} \to -1.034,\ \text{z3} \to -1.636\}$$

Justin's Tilt Test (per email of Mar 5 2008)

Requires case 20061213TM (T060283-02) to be loaded

```
Calculate[Stage2]
```

Start values for the minimization, all zero offset from equilibrium

```
sv = Table[allvars[[i]] -> 0, {i, Length[allvars]}];
```

■ Justin's case 1: 1 mN.m at top mass

```
{potential0, optval} = FindMinimum[
   allvars.potentialmatrix2.allvars - 0.001 * pitch0, Release[optspec[allvars, sv]]];

optval
```

$$\{x0 \to -1.15328 \times 10^{-7},\ y0 \to 0.,\ z0 \to 0.,\ yaw0 \to 0.,\ pitch0 \to 0.000115331,\ roll0 \to 0.,$$
$$x1 \to -3.13139 \times 10^{-7},\ y1 \to 0.,\ z1 \to 0.,\ yaw1 \to 0.,\ pitch1 \to 0.0000824822,\ roll1 \to 0.,$$
$$x2 \to -4.62611 \times 10^{-7},\ y2 \to 0.,\ z2 \to 0.,\ yaw2 \to 0.,\ pitch2 \to 0.0000669927,\ roll2 \to 0.,$$
$$x3 \to -5.94769 \times 10^{-7},\ y3 \to 0.,\ z3 \to 0.,\ yaw3 \to 0.,\ pitch3 \to 0.0000651715,\ roll3 \to 0.\}$$

■ Justin's case 2: 1 mN.m at UIM

```
{potential0, optval} = FindMinimum[
   allvars.potentialmatrix2.allvars - 0.001 * pitch1, Release[optspec[allvars, sv]]];

optval
```

$$\{x0 \to -8.24702 \times 10^{-8},\ y0 \to 0.,\ z0 \to 0.,\ yaw0 \to 0.,\ pitch0 \to 0.0000824774,\ roll0 \to 0.,$$
$$x1 \to -2.71223 \times 10^{-7},\ y1 \to 0.,\ z1 \to 0.,\ yaw1 \to 0.,\ pitch1 \to 0.00010628,\ roll1 \to 0.,$$
$$x2 \to -4.63818 \times 10^{-7},\ y2 \to 0.,\ z2 \to 0.,\ yaw2 \to 0.,\ pitch2 \to 0.0000863215,\ roll2 \to 0.,$$
$$x3 \to -6.34094 \times 10^{-7},\ y3 \to 0.,\ z3 \to 0.,\ yaw3 \to 0.,\ pitch3 \to 0.0000839715,\ roll3 \to 0.\}$$

■ Justin's case 3: 1 mN.m at PM

```
{potential0, optval} = FindMinimum[
   allvars.potentialmatrix2.allvars - 0.001 * pitch2, Release[optspec[allvars, sv]]];

optval
```

$$\{x0 \to -6.69877 \times 10^{-8},\ y0 \to 0.,\ z0 \to 0.,\ yaw0 \to 0.,\ pitch0 \to 0.0000669881,\ roll0 \to 0.,$$
$$x1 \to -2.20294 \times 10^{-7},\ y1 \to 0.,\ z1 \to 0.,\ yaw1 \to 0.,\ pitch1 \to 0.0000863189,\ roll1 \to 0.,$$
$$x2 \to -4.37615 \times 10^{-7},\ y2 \to 0.,\ z2 \to 0.,\ yaw2 \to 0.,\ pitch2 \to 0.000131002,\ roll2 \to 0.,$$
$$x3 \to -6.96051 \times 10^{-7},\ y3 \to 0.,\ z3 \to 0.,\ yaw3 \to 0.,\ pitch3 \to 0.000127435,\ roll3 \to 0.\}$$

■ Justin's case 4: 1 mN.m at optic

```
{potential0, optval} = FindMinimum[
   allvars.potentialmatrix2.allvars - 0.001 * pitch3, Release[optspec[allvars, sv]]];
```

```
optval
```

$$\{x0 \to -6.51648 \times 10^{-8},\ y0 \to 0.,\ z0 \to 0.,\ yaw0 \to 0.,\ pitch0 \to 0.0000651651,\ roll0 \to 0.,$$
$$x1 \to -2.14298 \times 10^{-7},\ y1 \to 0.,\ z1 \to 0.,\ yaw1 \to 0.,\ pitch1 \to 0.0000839682,\ roll1 \to 0.,$$
$$x2 \to -4.258 \times 10^{-7},\ y2 \to 0.,\ z2 \to 0.,\ yaw2 \to 0.,\ pitch2 \to 0.000127433,\ roll2 \to 0.,$$
$$x3 \to -7.12067 \times 10^{-7},\ y3 \to 0.,\ z3 \to 0.,\ yaw3 \to 0.,\ pitch3 \to 0.000159028,\ roll3 \to 0.\}$$

■ Justin's case 5: 1.5, 2, 2 and 1 mN.m on masses from top down

```
{potential0, optval} = FindMinimum[allvars.potentialmatrix2.allvars - 0.0015 * pitch0 -
    0.002 * pitch1 - 0.002 * pitch2 - 0.001 * pitch3, Release[optspec[allvars, sv]]];
optval
```

$$\{x0 \to -5.3709 \times 10^{-7},\ y0 \to 0.,\ z0 \to 0.,\ yaw0 \to 0.,\ pitch0 \to 0.00053709,\ roll0 \to 0.,$$
$$x1 \to -1.66706 \times 10^{-6},\ y1 \to 0.,\ z1 \to 0.,\ yaw1 \to 0.,\ pitch1 \to 0.000592884,\ roll1 \to 0.,$$
$$x2 \to -2.92251 \times 10^{-6},\ y2 \to 0.,\ z2 \to 0.,\ yaw2 \to 0.,\ pitch2 \to 0.000662557,\ roll2 \to 0.,$$
$$x3 \to -4.26465 \times 10^{-6},\ y3 \to 0.,\ z3 \to 0.,\ yaw3 \to 0.,\ pitch3 \to 0.000679585,\ roll3 \to 0.\}$$

Handy pitch frequency ready-reckoner

Needs case 20061213TM of Quad Xtra-Lite Lateral model to be loaded

In[53]:=
```
Calculate[Stage2]
```

Names of the ds

In[59]:=
```
ds = {dm, dn, d0, d1, d2, d3, d4};
```

The effective values

In[55]:=
```
eds = {0.001,0.001,0.001,0.001,0.001,0.001,0.001};
```

The masses creating torque

In[56]:=
```
ms = {mn3, mn3, m13, m13, m23, m23, m3} ;
```

Table of d name, effective value, mass, and net torque in N.m/rad

In[67]:=
```
Transpose[{ds, eds, ms/.constval,g*ms*eds/.constval}]//TableForm
```

Out[67]//TableForm=

dm	0.001	122.26	1.19937
dn	0.001	122.26	1.19937
d0	0.001	100.15	0.982473
d1	0.001	100.15	0.982473
d2	0.001	79.1392	0.776356
d3	0.001	79.1392	0.776356
d4	0.001	39.5696	0.388178

The total spring constant

In[68]:=
```
kk = Plus@@(g*ms*eds/.constval)
```

Out[68]=
```
6.30458
```

The moments of inertia

In[62]:=
```
{Iny,I1y,I2y,I3y}/.constval
```

Out[62]=
```
{0.0712, 0.0598, 0.420355, 0.420355}
```

The total moment of inertia

In[63]:=
```
II = Iny + I1y + I2y + I3y /. constval
```

Out[63]=
```
0.97171
```

The approximate pitch frequency (neglecting stretching of the wires)

In[69]:=
```
ff = Sqrt[kk/II]/2/Pi
```

Out[69]=
```
0.405397
```

Compare actual pitch frequency

In[76]:=
```
Hz2[[-1]]
```

Out[76]=
```
0.32341
```

The approximate pitch frequencies with an extra 0.001 of d at each level

In[74]:=
```
dff = Sqrt[(kk+g*ms*eds/.constval)/II]/2/Pi
```

Out[74]=
```
{0.44228, 0.44228, 0.435841, 0.435841, 0.429633, 0.429633, 0.41769}
```

The percentage change in pitch frequency for an extra 0.001 at each level

In[75]:=
```
Transpose[{ds,100*(dff/ff-1)}]//TableForm
```

Out[75]//TableForm=

dm	9.09804
dn	9.09804
d0	7.50976
d1	7.50976
d2	5.97837
d3	5.97837
d4	3.03256