Absolute Length Measurement of the Caltech 40 Meter Interferometer's Optical Cavities

Alberto Stochino, Koji Arai, Yoichi Aso, Rana Adhikari

Motivations

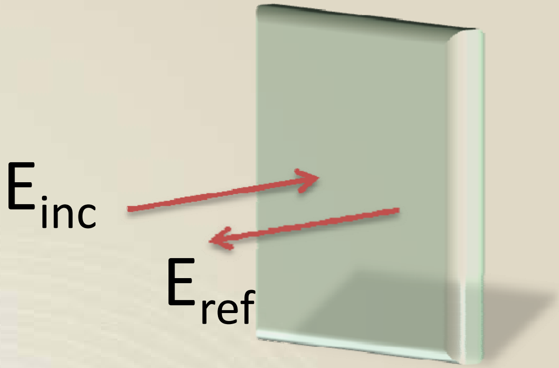
Why Size Does Matter

- **Cavity Characterization**
  - Optics metrology
  - Thermal Lensing Effects
- **Cavity and IFO More Accurate Modeling**
  - Cavity response to sidebands
  - Higher Order Mode Resonances Localization
  - Support to Lock Acquisition
- **Finer IFO Tuning**
  - Suspension Positioning
  - Sideband Frequencies
  - Demodulation Phases
- **Possible Effect of Length Detuning on DARM Noise**
  - Sideband Imbalance Induced by the Recycling Cavities
  - Sideband Intensity noise
  - Frequency noise

Interferometric Length Measurement

From the phase difference Φ between the incident and the returned light at distance L:

$$L = \frac{c}{2\nu}\left(\frac{\phi}{2\pi} + N\right)$$



N = # wavelengths in the round-trip optical path

An RF Modulated Field Makes easier to measure Φ and N for macroscopic distances L

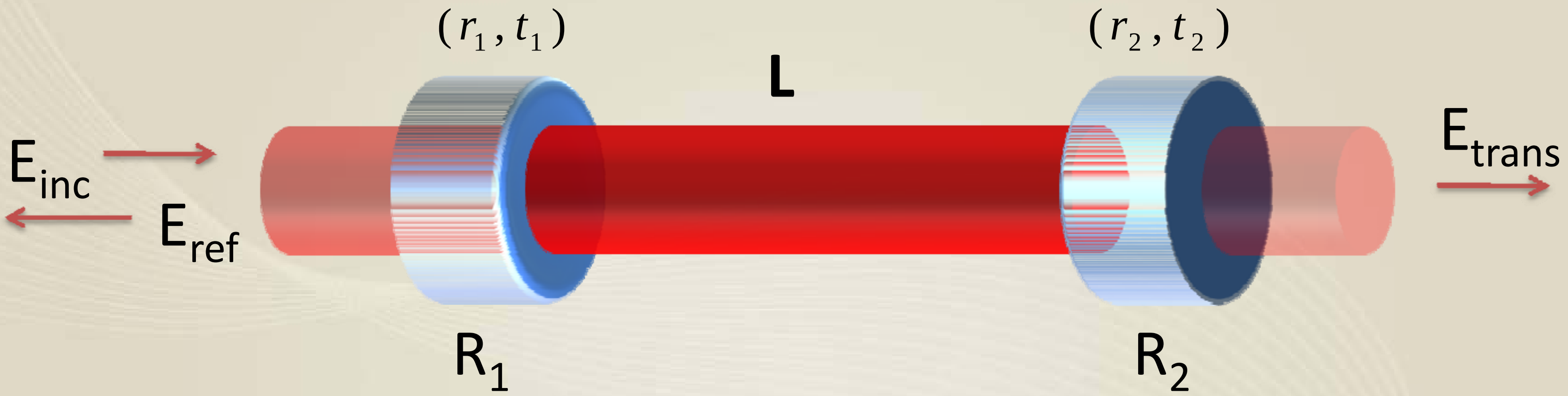
Resolution

$$\delta L = \frac{c}{2\nu_m}\frac{\delta\phi}{2\pi} = \frac{\lambda_m}{2}\alpha$$

For higher accuracy **Fabry-Perot Cavity**

Fabry-Perot Cavity

Fabry-Perot Cavity



TEM$_{00}$ Mode Resonances

$$\nu_n = \frac{c}{2L}\left\{ n + \frac{1}{\pi}\cos^{-1}\sqrt{\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)} \right\}$$

Free Spectral range

$$\nu_{FSR} = \nu_{n+1} - \nu_n = c/2L$$

Cavity Reflectance

$$r_{FP}(\omega) = \frac{E_r}{E_i} = \left[r_1 - \frac{t_1^2 r_2 \exp(-2i\omega L/c)}{1 - r_1 r_2 \exp(-2i\omega L/c)} \right]$$

FP Cavity Accuracy Enhancement

For ν_m near a cavity resonance:

$$\delta\phi = \Im[r_{FP}(\nu_m)] \approx \frac{r_2(1-r_1^2)}{(1-r_1 r_2)^2}\left(\frac{2L}{c}\right)2\pi\delta\nu_m$$

Since around the resonance $\delta\nu_m / \nu_m = \delta L / L$

$$\delta L = \underbrace{\frac{(1-r_1 r_2)^2}{r_2(1-r_1^2)}}_{\propto 1/\mathscr{F}}\left(\frac{c}{2\nu_m}\right)\frac{\delta\phi}{2\pi}$$

Cavity Finesse $\mathscr{F} = \dfrac{\pi\sqrt{r_1 r_2}}{1-r_1 r_2}$

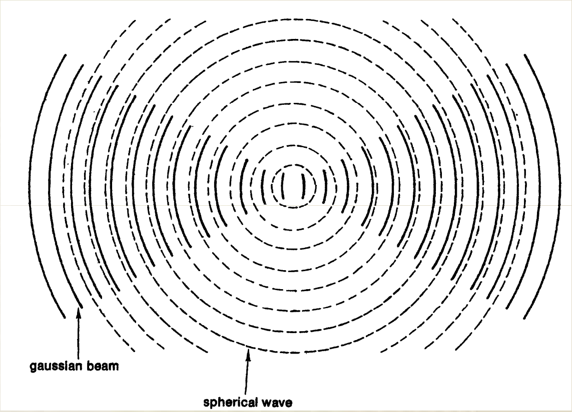
Transverse Mode Spacing

Beams: Hermite-gaussian representation

$$E(x,y,z)=\sum_{n,m}^{\infty}E_{nm}u_n(x,z)\times u_m(y,z)e^{ikz}$$

Phase Longitudinal Evolution

$$\Phi_{nm}(z)=kz+\phi_{nm}(z)$$



Guoy Phase

$$\phi_{nm}(z)=-i(n+m)[\psi(z)-\psi_0]$$

$$\psi(z)=\tan^{-1}(\frac{z}{z_R})$$

Resonant Condition in a Fabry-Perot Cavity

$$\Phi_{nm}(L)=\ell\pi$$

$$\nu_{\ell nm}=\frac{c}{2L}\left[\ell+(n+m+1)\frac{1}{\pi}\cos^{-1}\sqrt{\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)}\right]$$

Transverse Mode Spacing

$$\nu_{TMS}=\nu_{\ell,n,m}-\nu_{\ell,n,m-1}=\frac{1}{\pi}\cos^{-1}\sqrt{\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)}$$

g-factor

$$g_i=\left(1-\frac{L}{R_i}\right)$$

Absolute Length Measurements in GWID (1)

TAMA, JAPAN

A. Araya et al, Applied Optics 38 (1999) 2848-2856, "*Absolute-Length Determination of a Long-Baseline Fabry-Perot Cavity by Means of Resonating Modulation Sidebands*"

RF Phase Modulated field

$$E_{in} = E_0 e^{\gamma \cos(\omega_m t)} e^{i\omega t} \approx E_0 \left[J_0 e^{i\omega t} + iJ_1 e^{i(\omega - \omega_m)t} + iJ_1 e^{i(\omega + \omega_m)t} \right]$$

1. Cavity locked to both carrier and sidebands:
   - to the carrier with an auxiliary RF modulation frequency
   - to the sidebands by acoustic modulation of the sidebands and double demodulation PDH extraction
2. The PDH signal provides a way to measure the phase lag that one sideband accumulates inside of the cavity ν_m

$$\delta L / L = 3 \times 10^{-9}$$

Very accurate, but complex and not possible "online"

Absolute Length Measurements in GWID (2)

LHO 2k, 2000

FSR measured by tuning the sidebands frequency to complete anti-resonance when the carrier is locked.

(The anti-resonance is detected when a dip appears in the power spectrum at the AS port 's PD. A confirm comes from swinging one of the cavity mirror; the two sidebands' doublet fringes fade into only one). Accuracy 10^{-9}

- (B. Kells, elog 12/7/00; LIGO doc G010255-00)

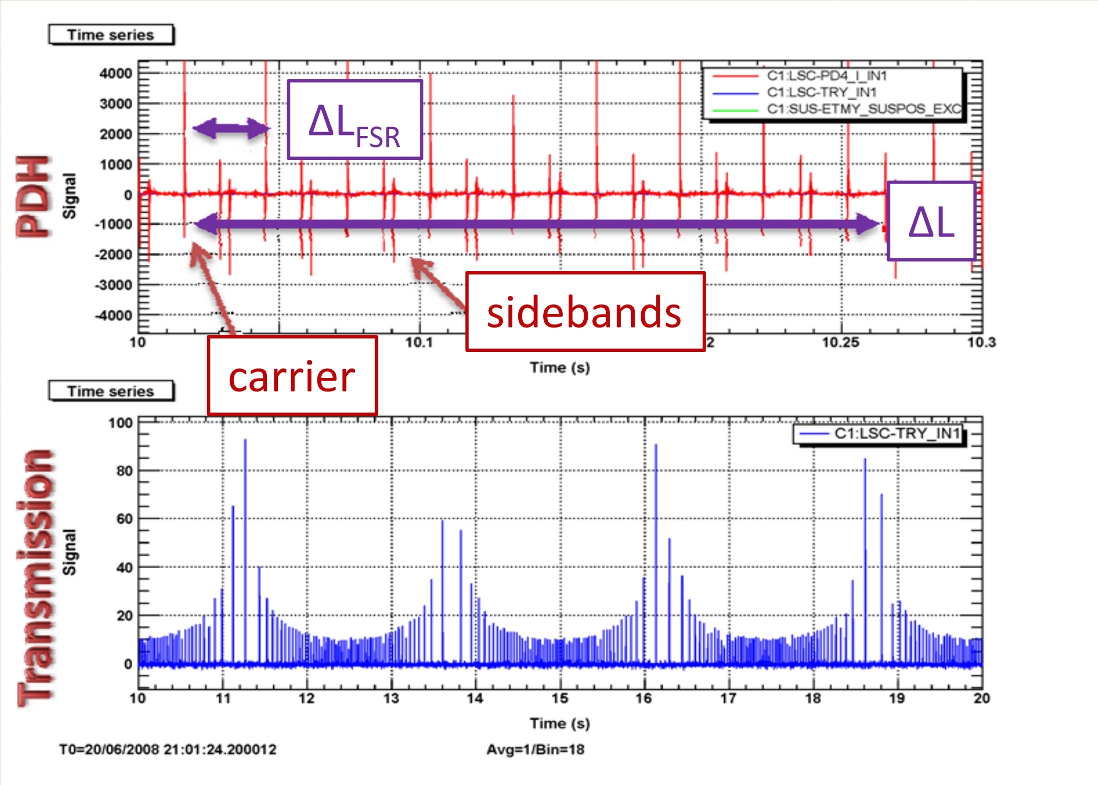
LHO 4k

Measurements of transfer functions by sweeping the sideband modulation frequency before the Mode Cleaner. Accuracy:

longitudinal mode spacing 2×10^{-8}, transverse mode spacing 2×10^{-8}

- M. Rakhmanov et al, Class. Quantum Grav. 21 (2004) S487-S492, "*Characterization of the LIGO 4 km Fabry.Perot cavities via their highfrequency dynamic responses length and laser frequency variations*"
- R. Savage et al, LSC Meeting on March 2005, LIGO document G050111-00, "*Summary of recent measurements of g factor changes induced by thermal loading in theH1 interferometer*"
- R. Savage et al, Poster in 6th Edoardo Amaldi Conference (2006), LIGO document G050362-00, "*Measurement of thermally induced test mass surface curvature changes in a LIGO 4-km interferometer*"

The Vernier Technique (3)



The cavity length is swept by exciting one mirror.

ΔL = distance between carrier and one sideband relative to the same longitudinal mode n

ΔL_{fsr} = distance between two adjacent longitudinal modes of the carrier

$$\frac{\Delta L}{\Delta L_{fsr}} \approx \frac{\nu_m}{\nu_{fsr}}$$

The cavity length is changing!

Not very accurate: ~ 10^{-3}.

"If I used my finger to measure the length, that would be as much accurate!"

Rana Adhikari, Caltech, circa August 2008

M Rakhmanov, M Evans and H Yamamoto, Meas. Sci. Technol. 10 (1999) 190–194. *"An optical Vernier technique for in situ measurement of the length of long Fabry–Perot cavities"*

A New Technique

A RF modulation is produced by the beating of the main beam with that of an auxiliary laser at a slightly different frequency.

Interference Field

$$E(x,t) = E\left[e^{i(\omega_1 t + k_1 x)} + e^{i(\omega_2 t + k_2 x)}\right]$$

$$\left(E_1 = E_2 = E \ \text{ for simplicity}\right)$$

Power Detection Measurement

ω_1 resonant

$$\Delta\omega = \omega_2 - \omega_1 = \mathrm{n}\omega_{\mathrm{FSR}}$$

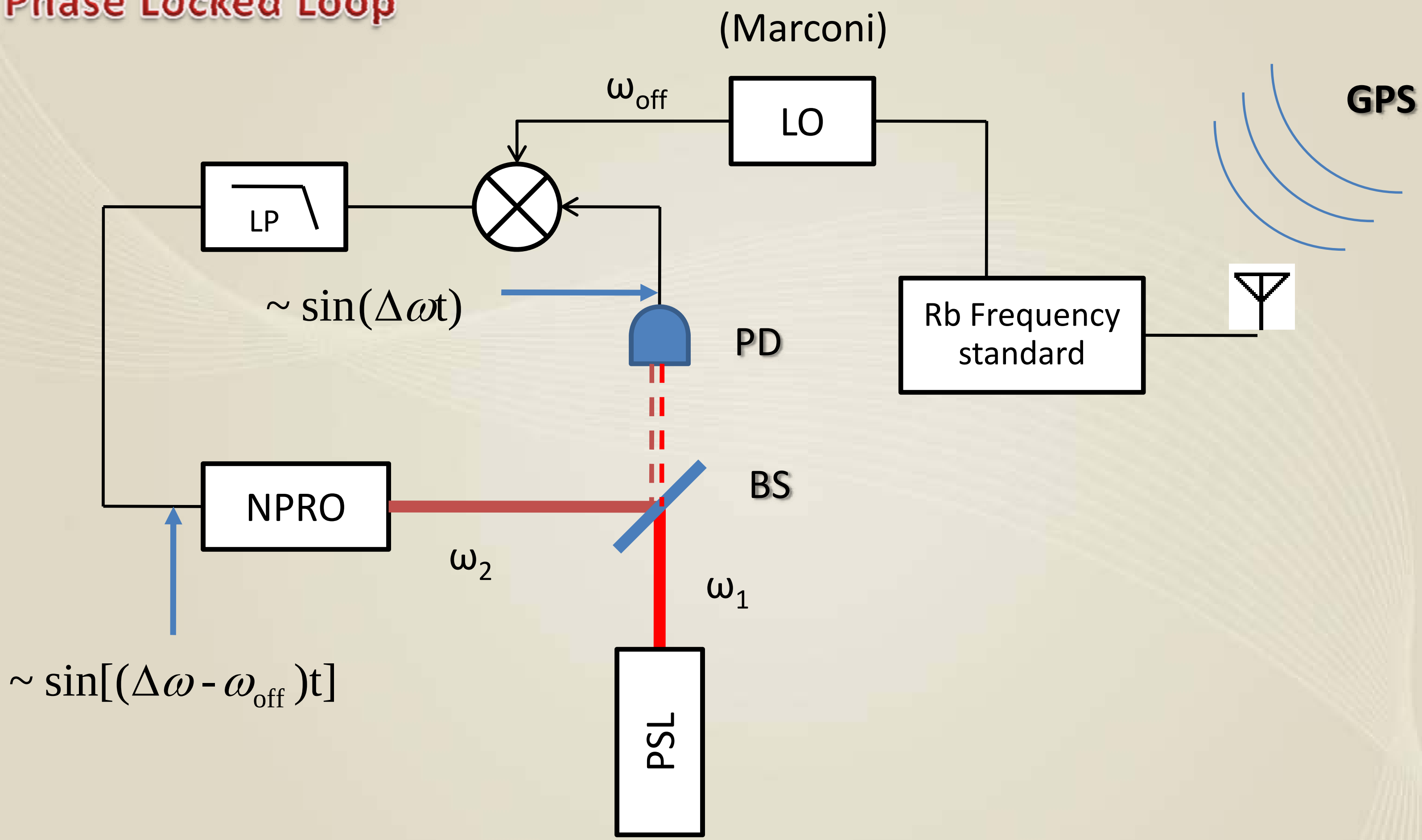
$$E_t = EE^* = 2TE^2\left[1 + \cos(\Delta\omega t)\right]$$

Conditions for the measurement:

- Cavity locked to main laser (ω_1)
- Auxiliary laser's frequency ω_2 locked to ω_1 by a tunable offset $\Delta\omega$

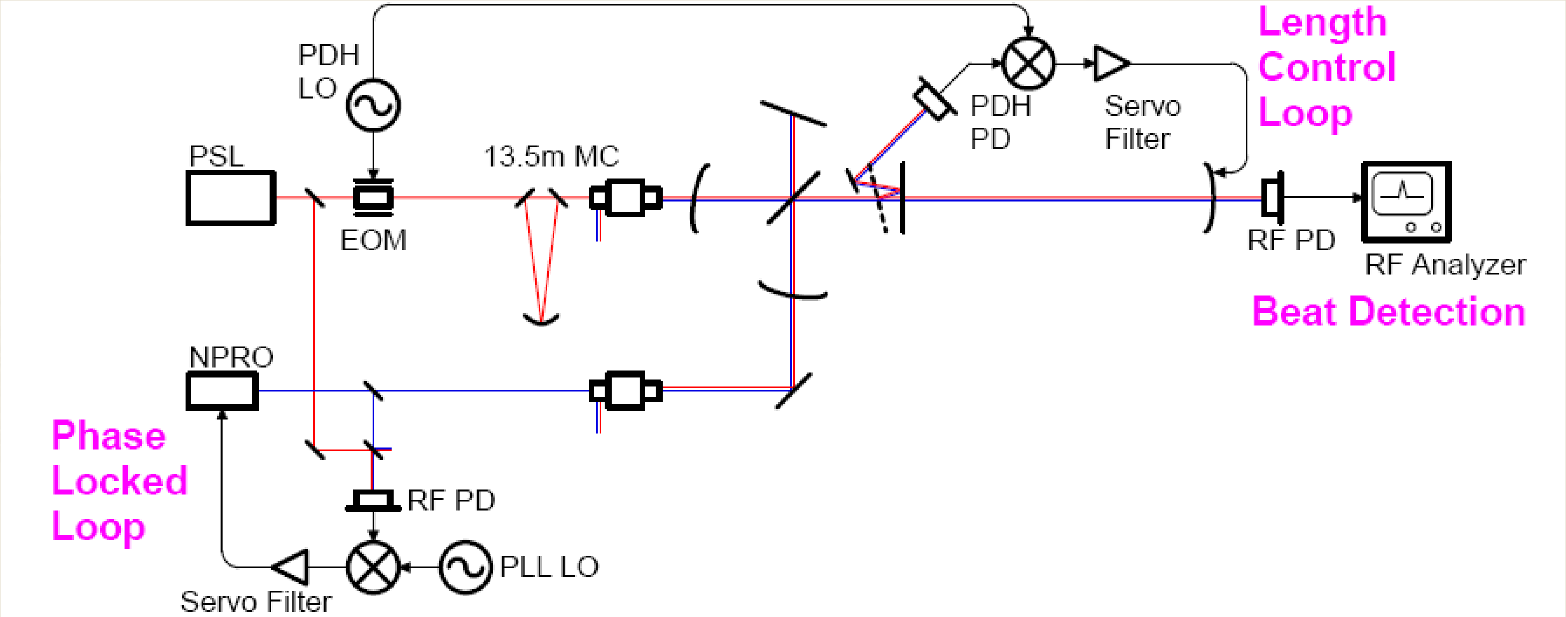
Locking the auxiliary laser to the PSL

Phase Locked Loop



James B.Armor, Jr., and Stanley R. Robinson, Applied Optics, Vol. 18 No. 18 (1979), "*Phase-lock control considerations for coherently combined lasers*"

Arm Measurement Scheme



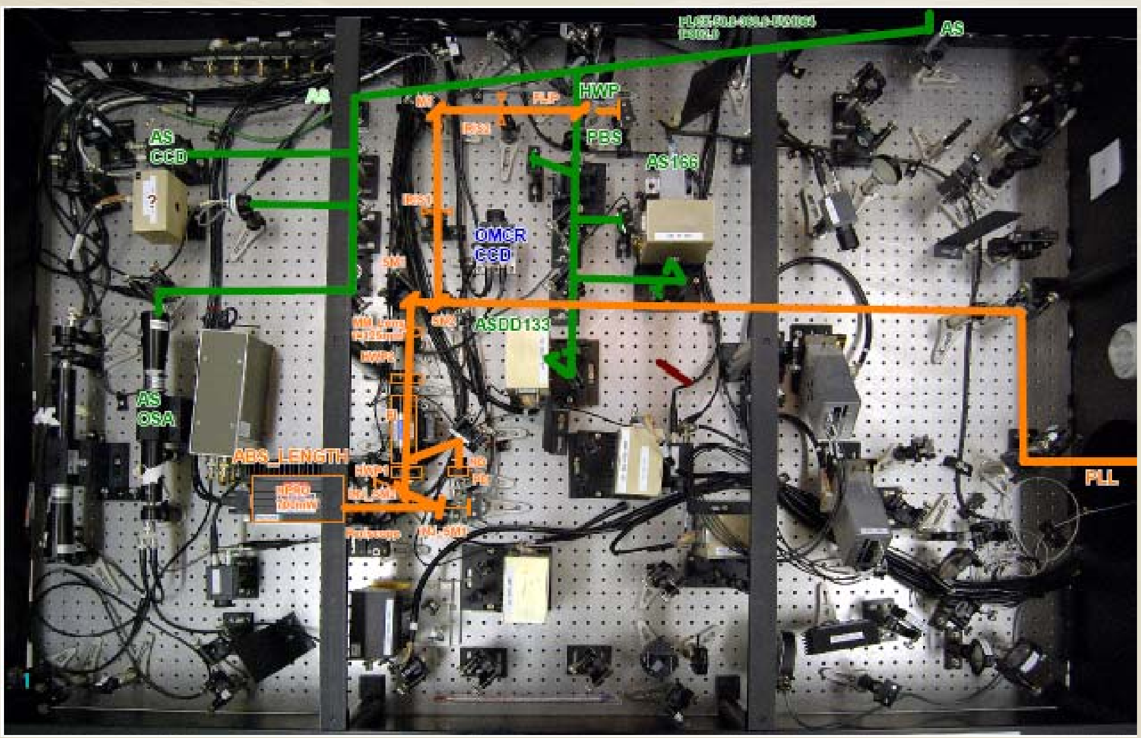
Strategy

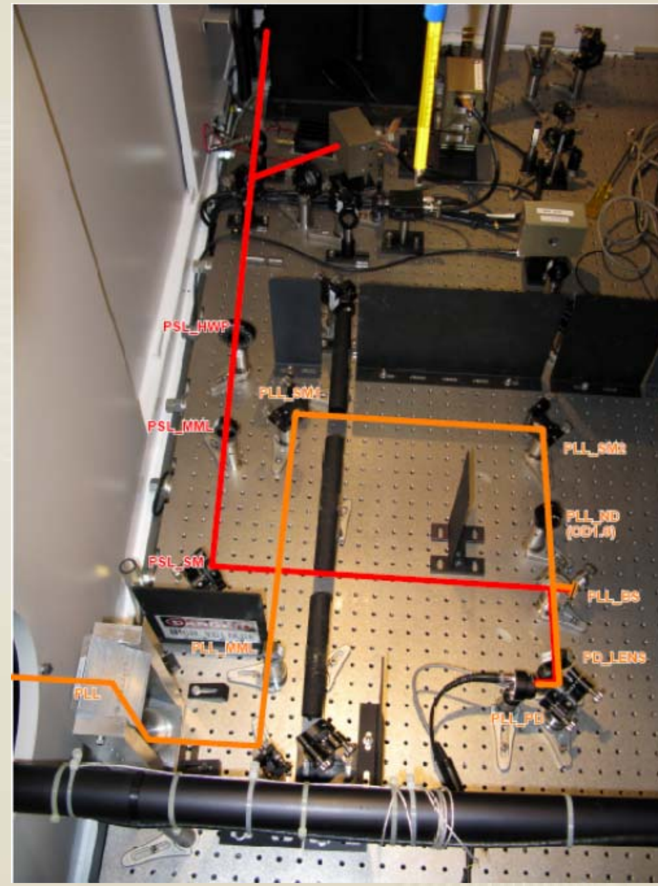
- Auxiliary beam injected from the dark port
- Cavity locked to the main beam
- Frequency difference of the two lasers stabilized by the PLL servo
- Beating appears at the transmission only when aux. beam is resonant
- Mode spacing read from the LO freq of the PLL at max of transmission

Optics Setup

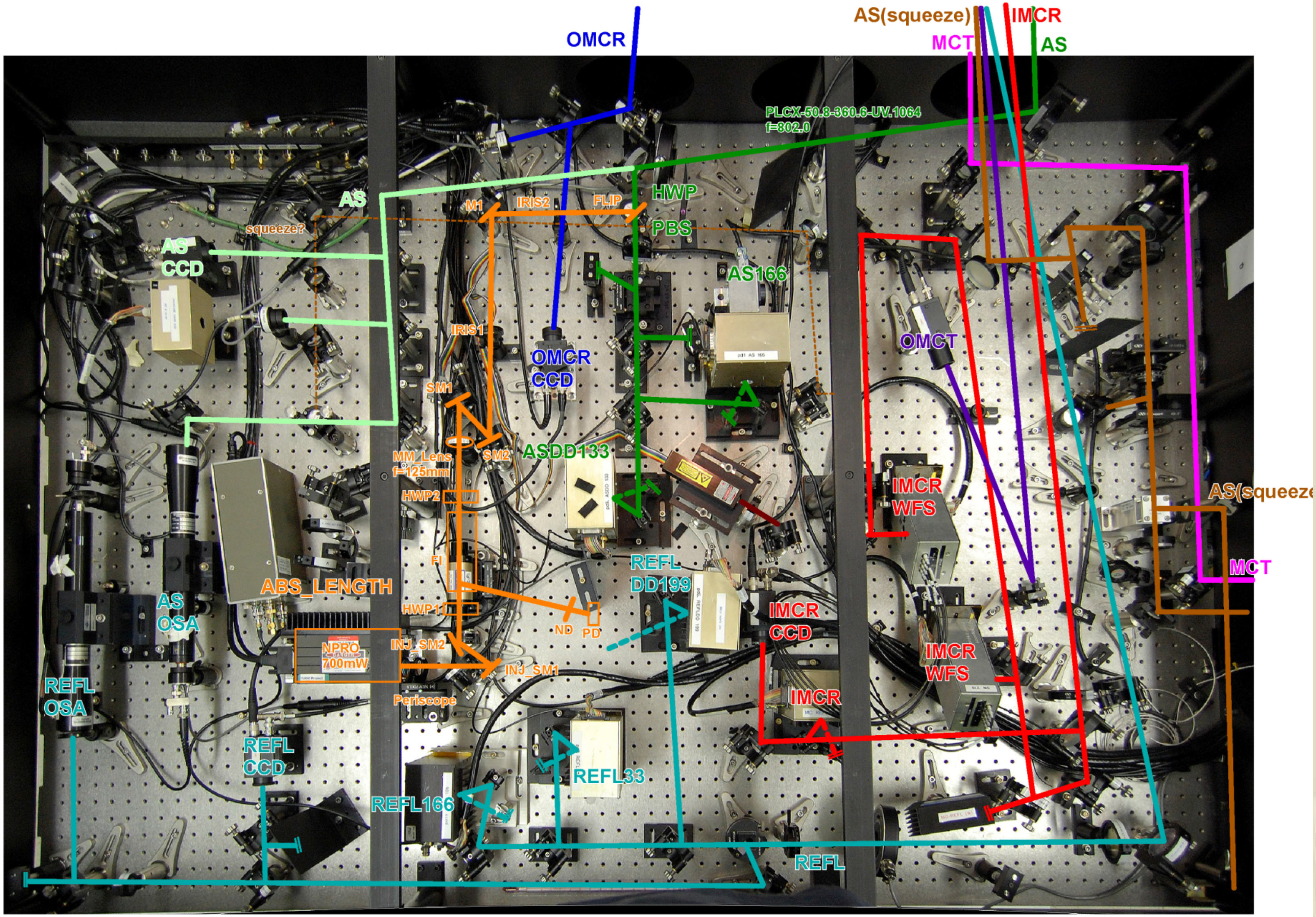
AS Table

AP Table

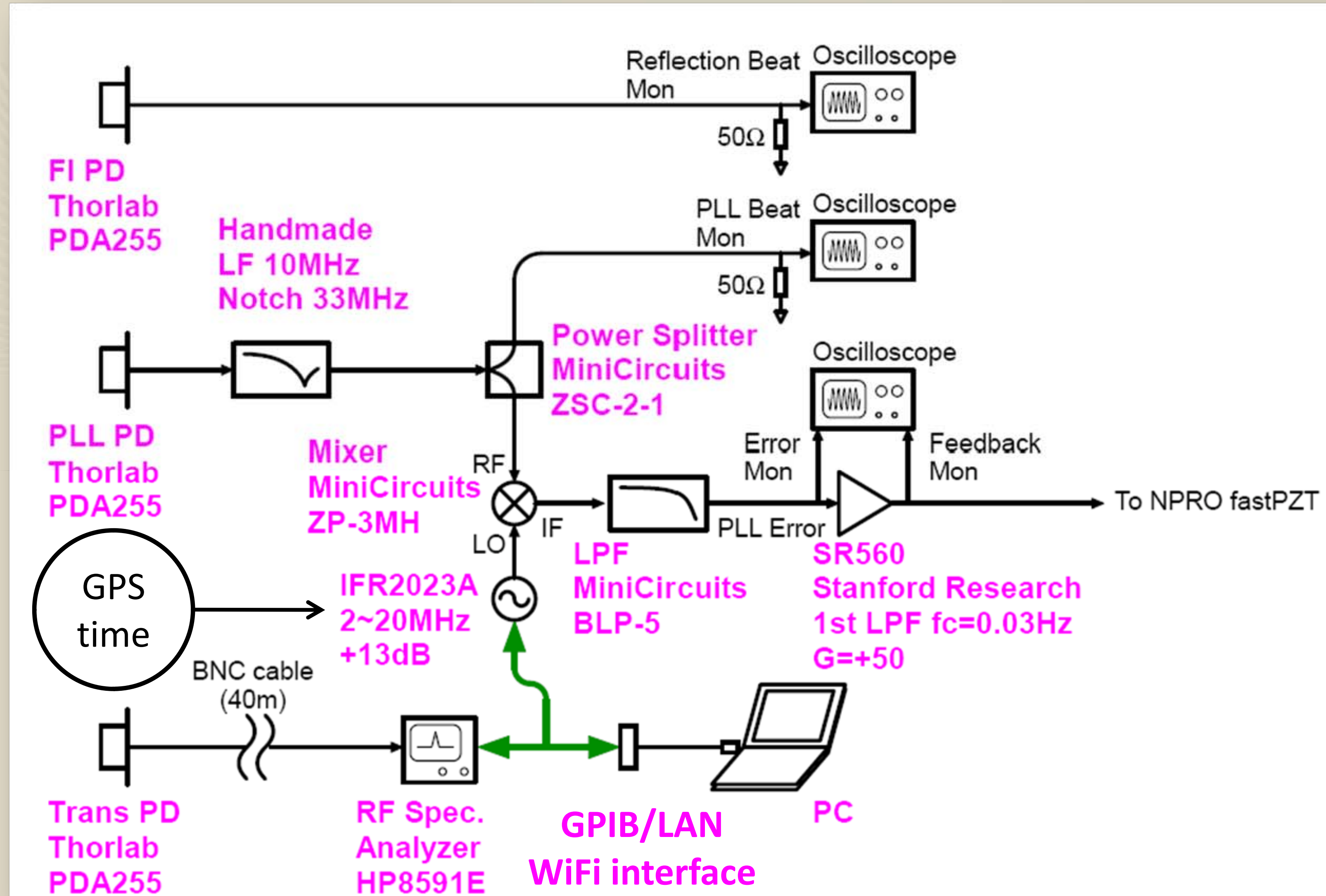




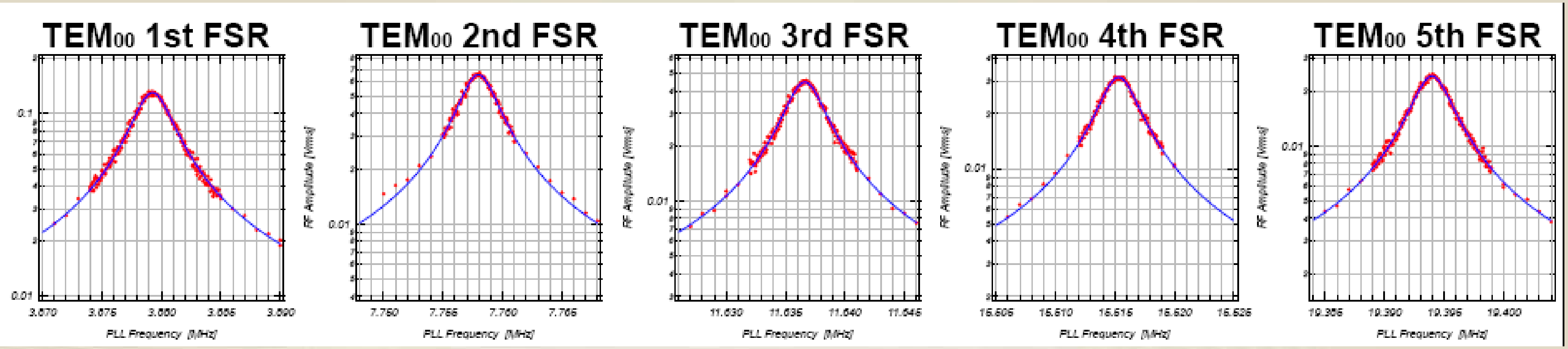
Injection Optics – AP Table Detail

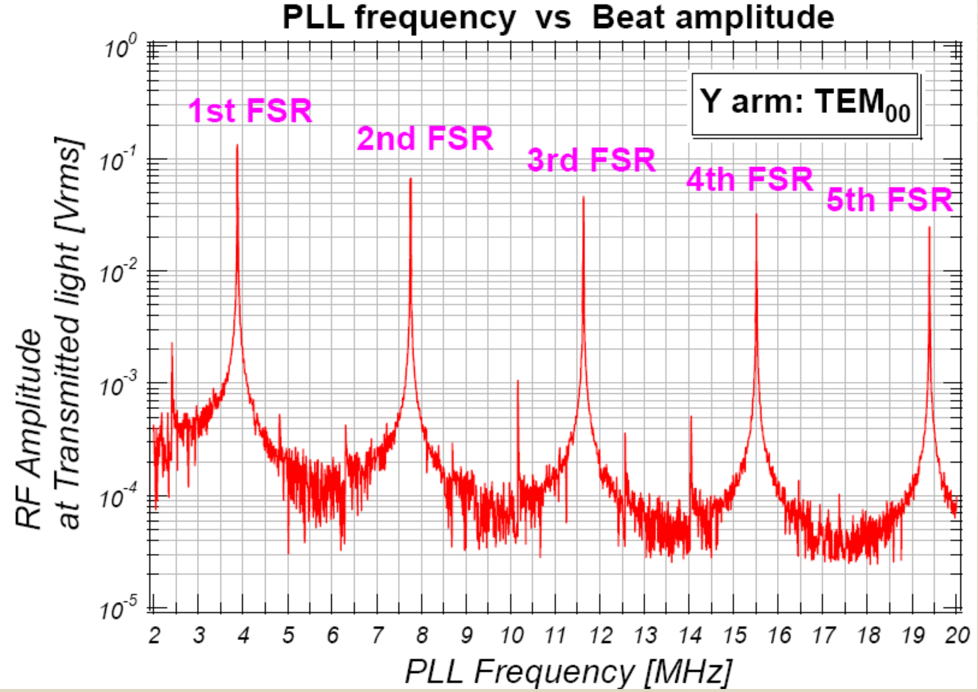


Detection Setup



X Arm FSR Series



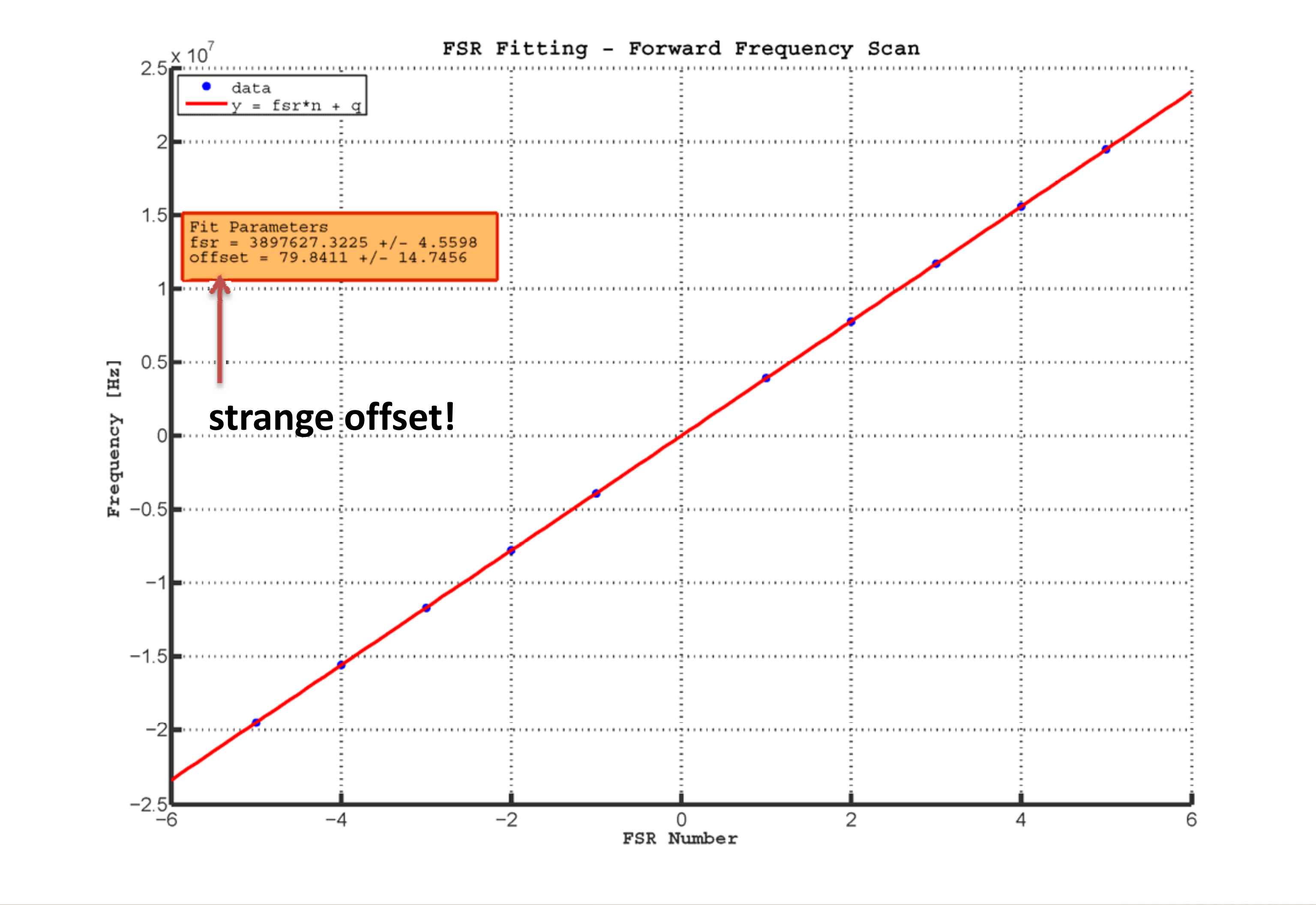


Fit

$$V(f) = \frac{A}{\sqrt{1 - \frac{(f - f_0)^2}{f_c^{\,2}}}}$$

FSR Fit

Y-Arm



FSR Fitting - Forward Frequency Scan
x 10^7
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
Frequency [Hz]
−6
−4
−2
0
2
4
6
FSR Number
data
y = fsr*n + q
Fit Parameters
fsr = 3897627.3225 +/- 4.5598
offset = 79.8411 +/- 14.7456
strange offset!

Mode Coupling

Mode Overlapping Ratio

$$E_1 = \sqrt{P_1} e^{i\omega_1 t} \left| TEM_{00} \right\rangle$$

$$E_1 E_1^* = P_1$$

$$E_2 = \sqrt{\alpha P_2} e^{i\omega_2 t} \left| TEM_{00} \right\rangle + \sqrt{(\alpha - 1) P_2} e^{i\omega_2 t} \left| TEM_{xx} \right\rangle$$

$$E_2 E_2^* = P_2$$

If $\nu_2 = \nu_1 + n\nu_{FSR}$ at the transmission: $E_2 = \sqrt{\alpha P_2} e^{i\omega_2 t} \left| TEM_{00} \right\rangle$

$$P_{beat,00} = (E_1 + E_2)(E_1 + E_2)^* = P_1 + P_2 + 2\sqrt{\alpha P_1 P_2} \cos(\Delta\omega t)$$

TEM_{01} / TEM_{10}

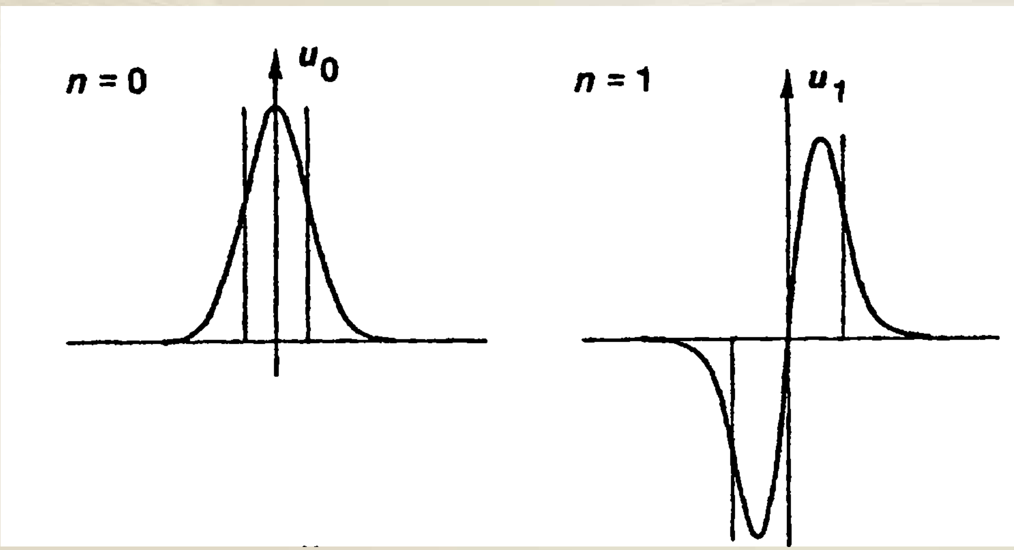
If $\nu_2 = \nu_1 + \frac{1}{\pi} \cos^{-1} \sqrt{\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)} \frac{c}{2L}$ at the transmission: $E_2 = \sqrt{(\alpha - 1) P_2} e^{i\omega_2 t} \left| TEM_{10} \right\rangle$

$$P_{beat,10} = (E_1 + E_2)(E_1 + E_2)^* = P_1 + (\alpha - 1) P_2 + 2\sqrt{\alpha(\alpha - 1) P_1 P_2} \cos(\Delta\omega t)$$

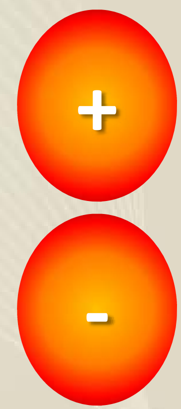
Transverse Beating Pattern

Transverse Beat

- The arm cavity is locked to the TEM_{01}/TEM_{10} of the main beam by tilting the End Mirror
- The beating with the TEM_{00} of the aux beam is visible in transmission



TEM_{01} phase

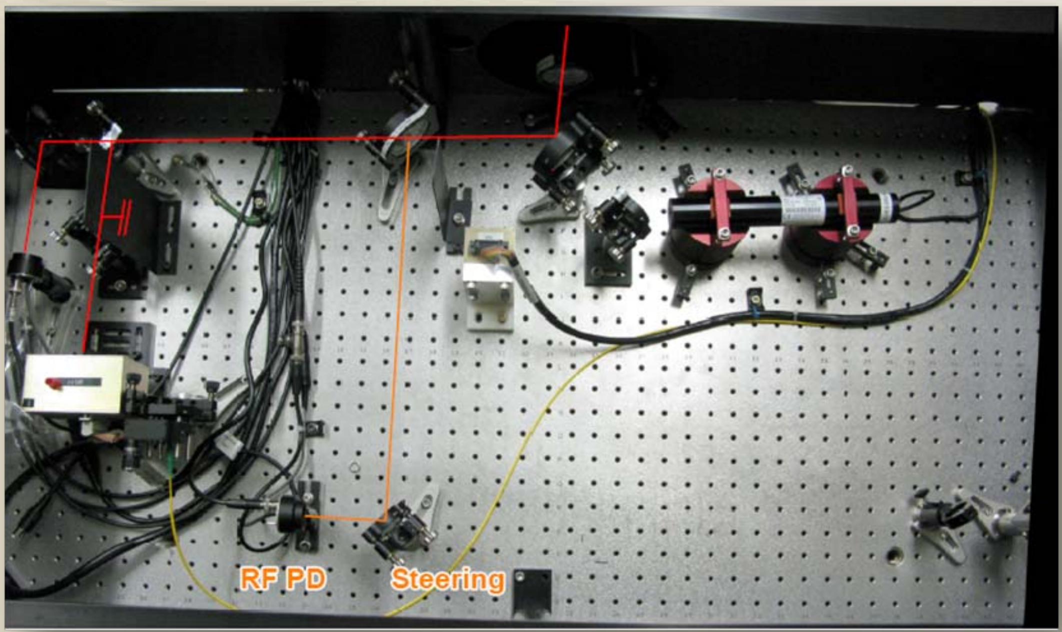


The lobes have opposite phase and the power hitting a photodiode is constant.
The beat does is not detected by a photodiode.

Shaving the Beam

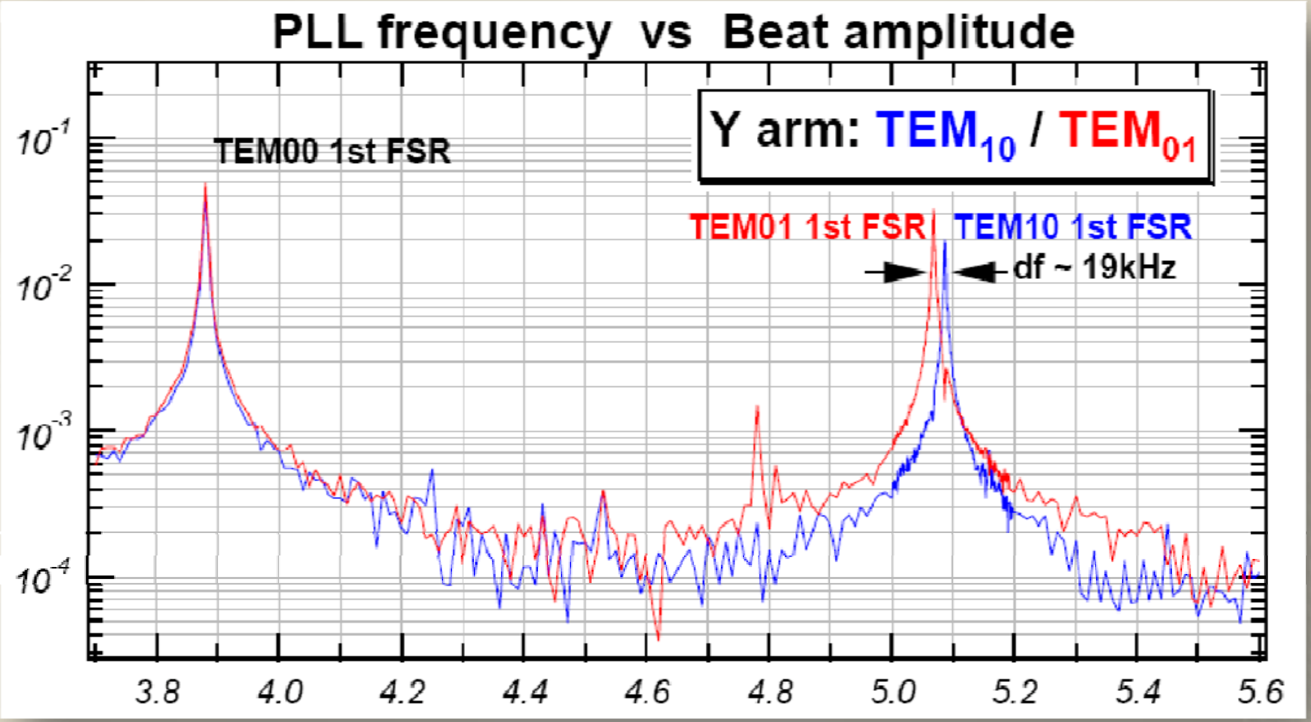
Optics Setup on the End Table

Knife edge place right in front of the PD to avoid diffraction



Astigmatic Mirrors

Y Arm Transverse Mode Spacing



The End Mirror's Astigmatism brakes the degeneracy of the 10 and 01 modes

$$R_1 = \infty \qquad \nu_{10} - \nu_{01} = \frac{1}{\pi}\left[\cos^{-1}\sqrt{1-\frac{L}{R_{2x}}} - \cos^{-1}\sqrt{1-\frac{L}{R_{2y}}}\right]$$

Summary of Measurements on the Arm Cavities

X Arm

FSR = (3897627 +/- 5) Hz
L = (38.45833 +/- 0.00005) m
g_{2x} = 0.31197 +/- 0.00004
g_{2y} = 0.32283 +/- 0.00004
$R_{ETM,x}$ = (55.8957 +/- 0.0045) m
$R_{ETM,y}$ = (56.7937 +/- 0.0038) m

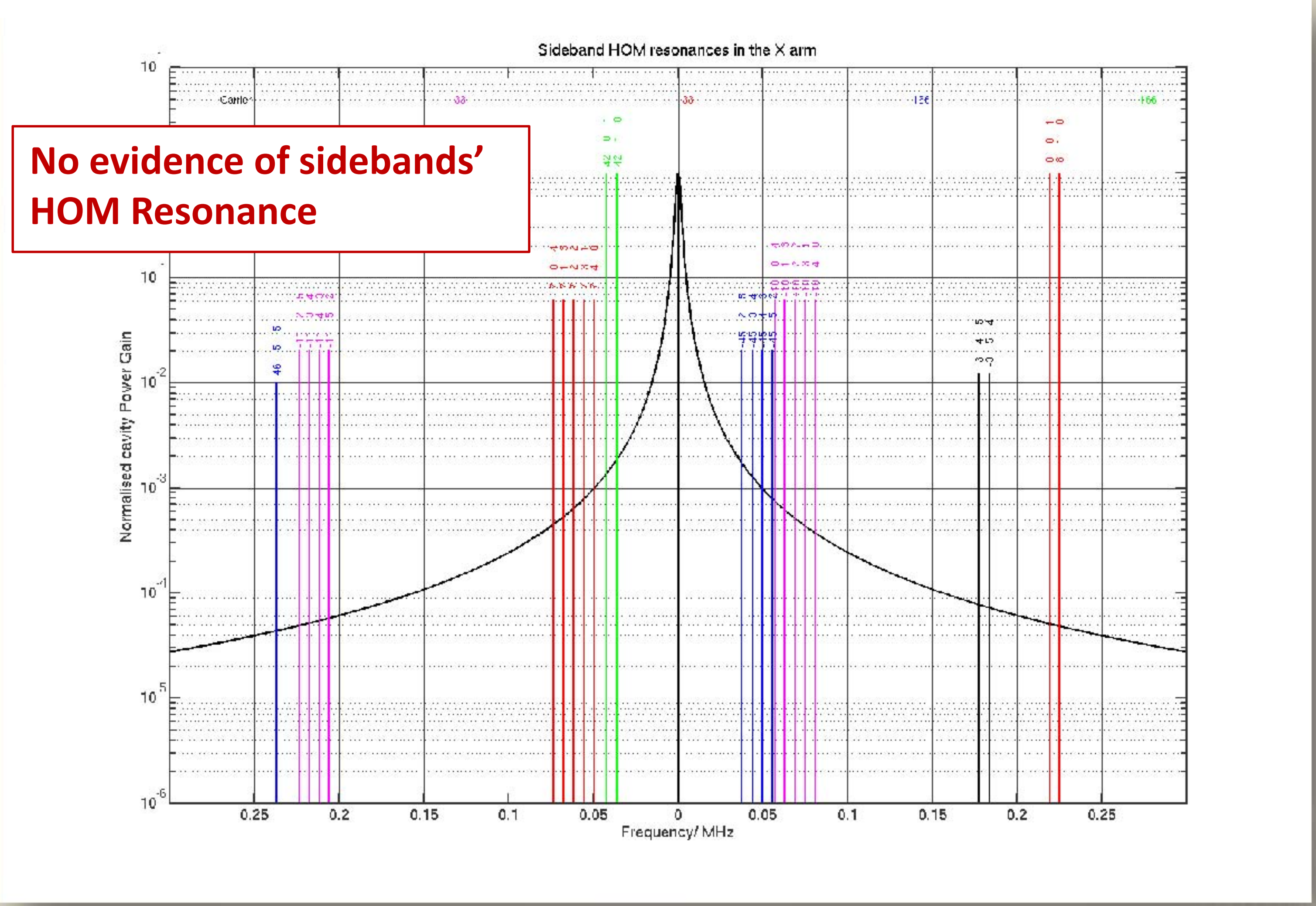
Y Arm

FSR = (3879252 +/- 30) Hz
L = (38.6462 +/- 0.0003) m
g_{2x} = 0.31188 +/- 0.00004
g_{2y} = 0.32601 +/- 0.00004
$R_{ETM,x}$ = (56.1620 +/- 0.0013) m
$R_{ETM,y}$ = (57.3395 +/- 0.0011) m

LIGO's Metrology

(57.37 +/- 0.6) m

Mode Resonances at the 40m

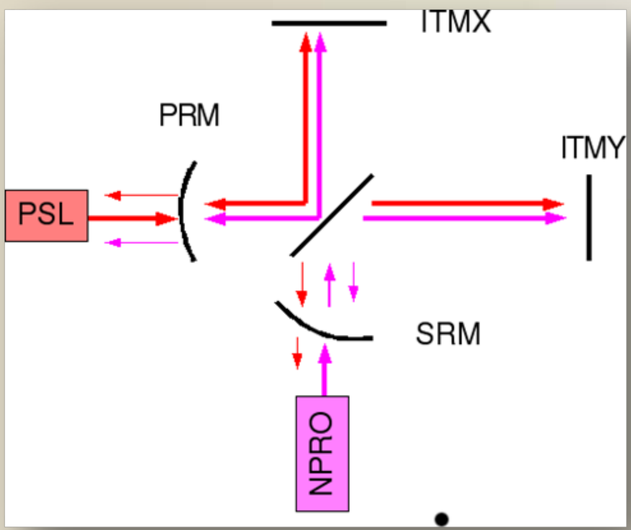


No evidence of sidebands' HOM Resonance

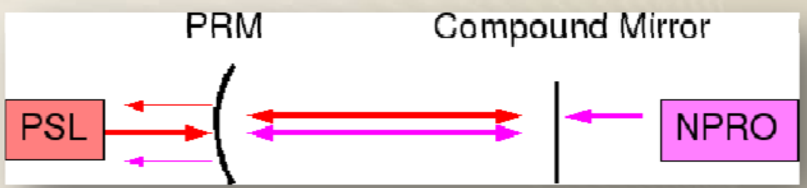
(Matlab code by J. Miller)

Recycling Cavities

- The finesse is lower (<80) and frequency dependent
  - Cavity modeling necessary
  - But it is easier to make the aux beam go through
- Where to inject the aux beam? The Schnupp Asymmetry makes things harder
  - Maybe not to much since it is frequency dependent
  - Doable for the PRC with current 40m configuration
  - SRC?



Effective Michelson Mirror



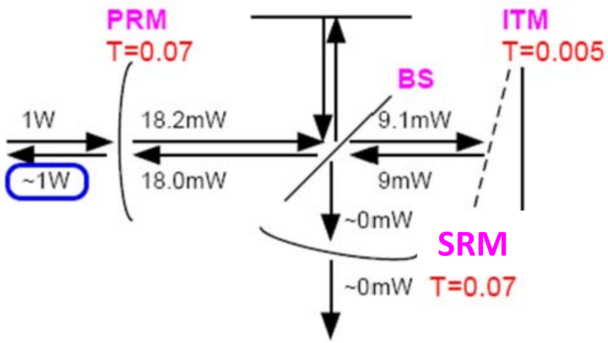
$$r_{com} = (-r_{BS})^2 r_1 e^{-i2l_1\Omega/c} + t_{BS}^2 r_2 e^{-i2l_2\Omega/c}$$

$$t_{com} = -r_{BS} t_{BS} r_1 e^{-i2l_1\Omega/c} + r_{BS} t_{BS} r_2 e^{-i2l_2\Omega/c}$$

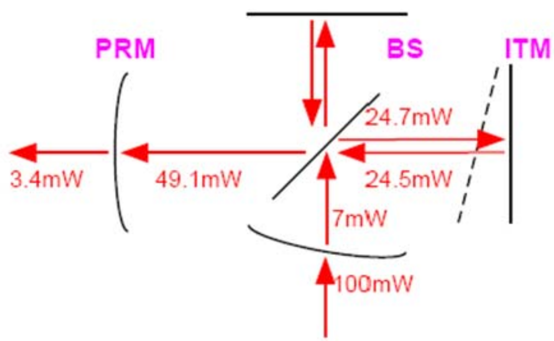
PRC Model

Short cavity length measurement

A) for the PSL beam (β0=π, Carrier Anti-resonant Case)



B) for the auxiliary beam



$P_{aux} / P_{PSL} = 0.0034$

Expected Max Contrast: $1-P_{min}/P_{max}$
$= 4\,(P_{aux}\,P_{PSL})^{1/2}/(P_{aux}^{1/2}+P_{PSL}^{1/2})^2$
$= 0.21$

Intracavity Power

$$P_{PRC}(\Omega) = \left|G_{PRC}(\Omega)\right|^2 \left|E_{in}\right|^2$$

$$G_{PRC} = \frac{r_{ITM}\,\sin(\alpha) e^{-i\delta\beta} e^{i l_{RM}\Omega/c}}{1 - r_{RM}\, r_{ITM}\, \cos(\alpha) e^{-i\delta\beta}}$$

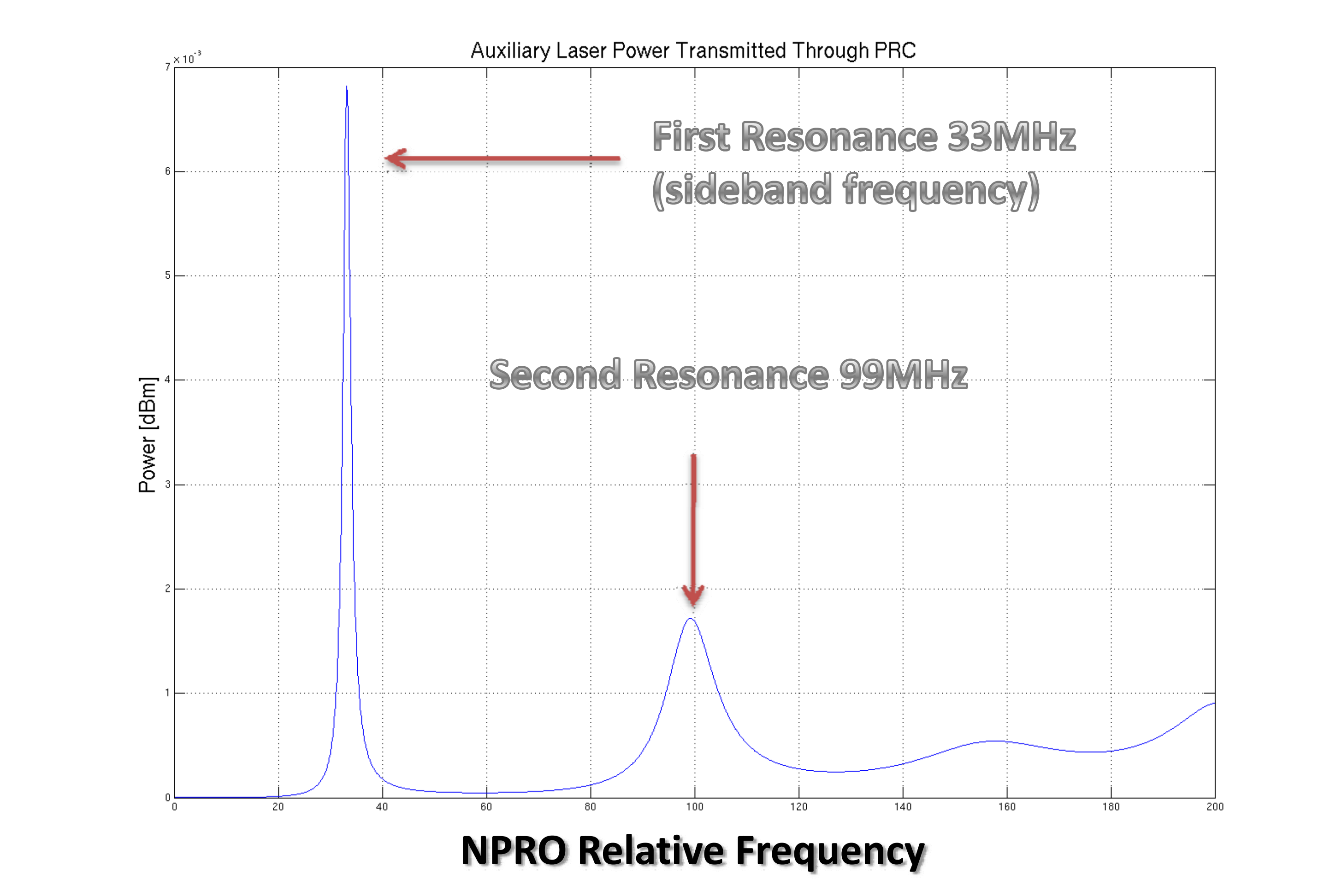
$$\Omega = \Omega_0 + \omega$$

$$\alpha = \frac{\Delta l \Omega}{c}$$

$$\beta = \frac{(l_1 + l_2 + 2 l_{RM})\Omega}{c} = \frac{2 l_{PRC\Omega}}{c}$$

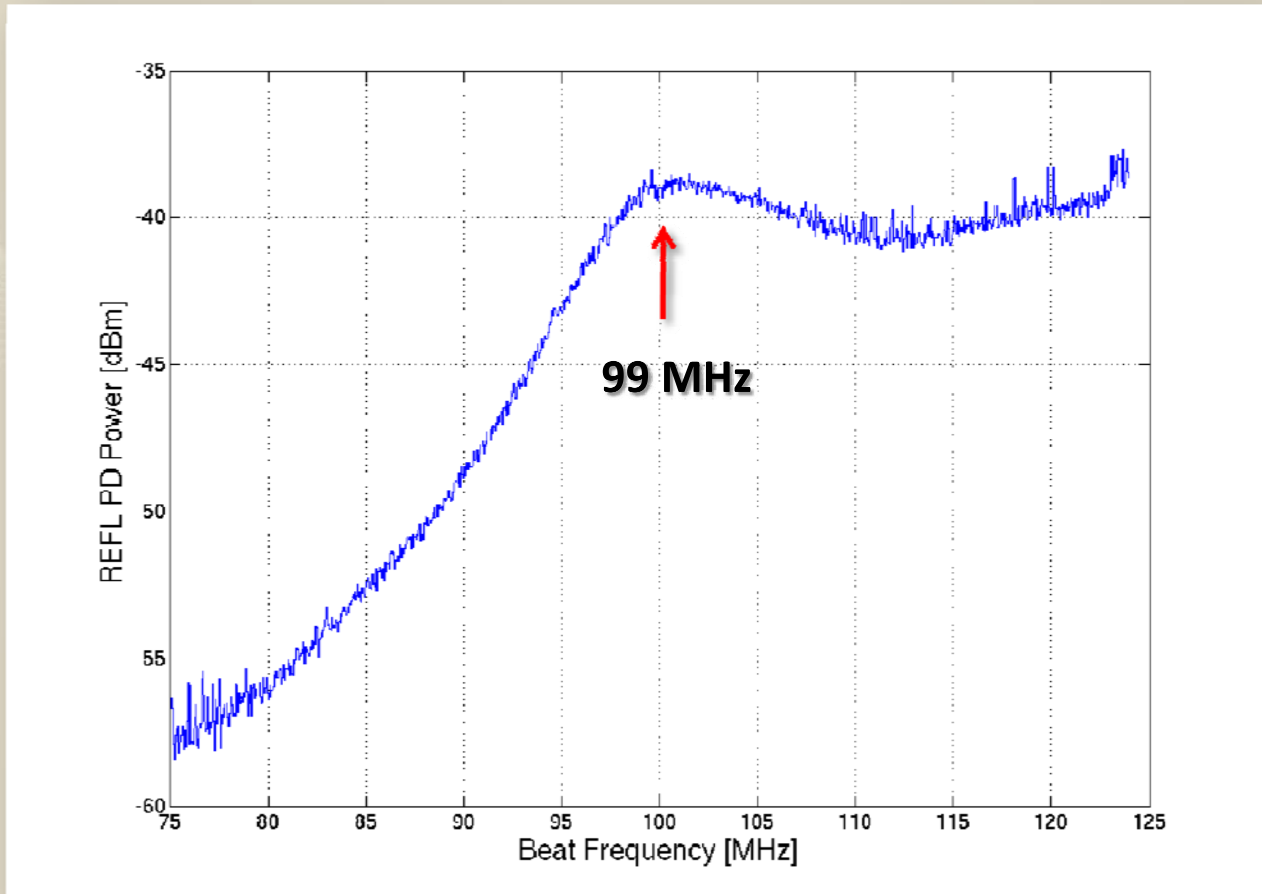
PRC Expected Intra-cavity Power

Carrier Anti-resonant, nominal Schnupp Asymmetry



PRC Preliminary Results

PRM Reflection PD



Adjustment of the model parameters necessary to extract the length from the measurement

Future Work

- **Continue the measurements on PRC**
  - Get a new faster PD (no filter, larger bandwidth)
  - Turn off f2
  - Fit data with model
- **Measure SRC**
  - how to go through MC?
- **Phase detection instead of power detection?**
  - The aux beam is not stable enough because the low gain/bandwidth of the PLL servo
  - The phase of the NPRO is locked, but not its frequency. The beam has arbitrary phase
- **Study effect on Advanced LIGO Sensitivity**
  - Modeling Frequency and Amplitude noise on DARM