A Bayesian stack-slide search for X-ray pulsations from LMXBs

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We describe a new semi-coherent Bayesian search strategy designed to analyse the RXTE data archive with the aim of searching for pulsations from the galactic low-mass X-ray binaries for which pulsations have not yet been found. This method follows a Bayesian approach to the standard stack-slide search strategy and is potentially ~ 5 times more sensitive to pulsations than previous analyses. This is because all archive observations are semi-coherently combined into the search and in the event of a non-detection the Bayesian approach allows upper-limits on the fractional amplitude of the pulsations to be output directly. The primary motivation behind this analysis is to obtain accurate spin and orbital parameter values for these objects such that gravitational wave searches may benefit from the parameter space volume reduction.

▸ Introduction

The absence of pulsations in the X-ray emission of the galactic low-mass X-ray binaries (LMXBs) has been the cause of much debate. Since the Rossi X-ray Timing Explorer (RXTE) was pointed at the brightest of these objects, Sco X-1, at the start of it's operation in 1996 and failed to detect pulsations various theories have emerged to explain the possible weakness of such a signal. The possibility of reduced pulsation amplitude has been attributed to gravitational lensing [?] effects, Scattering in the optically thick neutron star (NS) environment [?], and the reduction of the NS magnetic field through screening due to the continual accretion of mass [?].

From a NS evolution perspective the spin period is arguably the most important parameter to determine in these systems. Millisecond period X-ray pulse detection would add weight to the proposal that LMXBs are the progenitors of the millisecond radio pulsars. It would also make it possible to determine orbital characteristics and the NS masses.

In terms of gravitational wave (GW) detection, LMXBs provide an excellent candidate signal source. However, despite their relatively strong predicted amplitude, they require knowledge of the time-dependent signal waveform to high accuracy in order to process the gravitational wave data optimally. Unfortunately, the system parameters defining the waveform include the orbital and spin parameters of the NS and for LMXBs these are generally poorly constrained, especially the spin parameters [?]. Determination of the spin frequency of the LMXBs would *significantly* increase the probability of the detection of GWs from these objects.

Existing upper limits [?, ?] on the modulation depth (pulse fraction) of a number of LMXBs are in the range $0.3\% - 5.0\%$. We predict that the search method described here will allow us to probe the 0.1% modulation depth using existing RXTE archival data.

▸ A Bayesian approach to stack-slide

The standard stack-slide search used extensively in EM and GW astronomy [?] starts with dividing the data into short segments. The phase-maximised Fourier power is then computed for each segment over a range in frequency and finally multiple trials are performed whereby the power is summed according to the frequency evolution of the source as predicted by a signal model. This is an incoherent search method and as such is less sensitive but also less computationally intensive than a corresponding coherent search method.

The search we describe here is a Bayesian approach to the standard stack-slide method for X-ray data. The basic difference being that instead of summing the *phase-maximised* likelihood (the power) from each segment we compute the product of the analytically *phase-marginalised* likelihood. This leaves us with

$$L(x|\mathbf{\Lambda}) \propto \int_0^{2\pi} \ldots \int_0^{2\pi} \prod_{j=1}^{M} L_j(x|\mathbf{\Lambda}, \phi_j) d\phi_1 \ldots d\phi_M \qquad (1)$$

where M is the number of segments, x is the data, ϕ_j are the initial phases of the signal for each segment, and $\mathbf{\Lambda}$ is a vector containing the remaining signal parameters. The quantity L is the likelihood on the signal parameter space where we have assumed that we know nothing about any of the ϕ_j values.

In order to obtain posterior probability distribution functions (PDFs) on the remaining $\mathbf{\Lambda}$ parameters one simply marginalises the product of L and the prior PDFs on $\mathbf{\Lambda}$ appropriately. A schematic view of this method is shown in fig. 1.

▸ Application to RXTE data

Let our signal model be

$$s_{ij} = A\Delta t \sin \Phi_{ij} + r_j \Delta t, \qquad (2)$$

where A is the pulsation amplitude count rate, r_j is the background count rate, and Δt is the sampling time of our data. We use i to index over the time within each segment and j to index the segments. Our binary phase model is approximated by the following linear model :

$$\Phi_{ij} \approx \phi_j + 2\pi \left(\nu t_i + \dot{\nu} t_i^2 + \ldots \right). \qquad (3)$$

Since we are limited to analysing short ~ 1000 second segments of data (typical of low Earth orbit satellites) and the orbital periods of most of the non-pulsating LMXBs lie in the range of hours to days we find that a 2^{nd} order expansion is typically sufficient.

The time binned data from RXTE is drawn from a Poisson distribution and in the regime $A \ll r$ and after marginalisation over the initial phase then we obtain

$$L_j(\mathbf{\Lambda}) = e^{-A^2 T_j/4r_j} I_0 \left(\frac{A}{r_j} |\tilde{x}_j(\nu, \dot{\nu})| \right) \qquad (4)$$

where I_0 is the modified Bessel function of the first kind and $\tilde{x}_j(\nu, \dot{\nu})$ is the Fourier transform of the data after re-sampling in time to correct for the frequency and frequency derivative parameters.

The final and most computationally challenging procedure is to compute the numerical marginalisation of the product of the segment likelihoods. Underlying the marginalisation is the appropriate sampling of both the individual segment likelihoods in the $\nu, \dot{\nu}$ space and the final combined likelihood on the physical parameters of the system. In order to perform correct covering in both cases likelihood values are sampled according to parameter spacings given by the metric on the log-likelihood. The metric $g_{\alpha\beta}$ is defined by

$$\mu = E \left[\frac{\log L_0 - \log L}{\log L_0} \right] = g_{\alpha\beta} \Delta\lambda^\alpha \Delta\lambda^\beta \qquad (5)$$

where L_0 represents the likelihood evaluated at the true signal parameters, the mismatch μ is the fractional loss in log-likelihood (also therefore proportional to the loss in signal amplitude) and $\Delta\lambda^\alpha$ represents deviations in the α'th signal parameter from the true values.

For both the segment and final likelihoods hyper-cubic grids are used in the sampling in order to allow for simple and fast numerical marginalisation, the result of which being posterior PDFs on the signal parameters (see fig. 2 for example results). The most important from a detection perspective being the PDF on the signal amplitude.

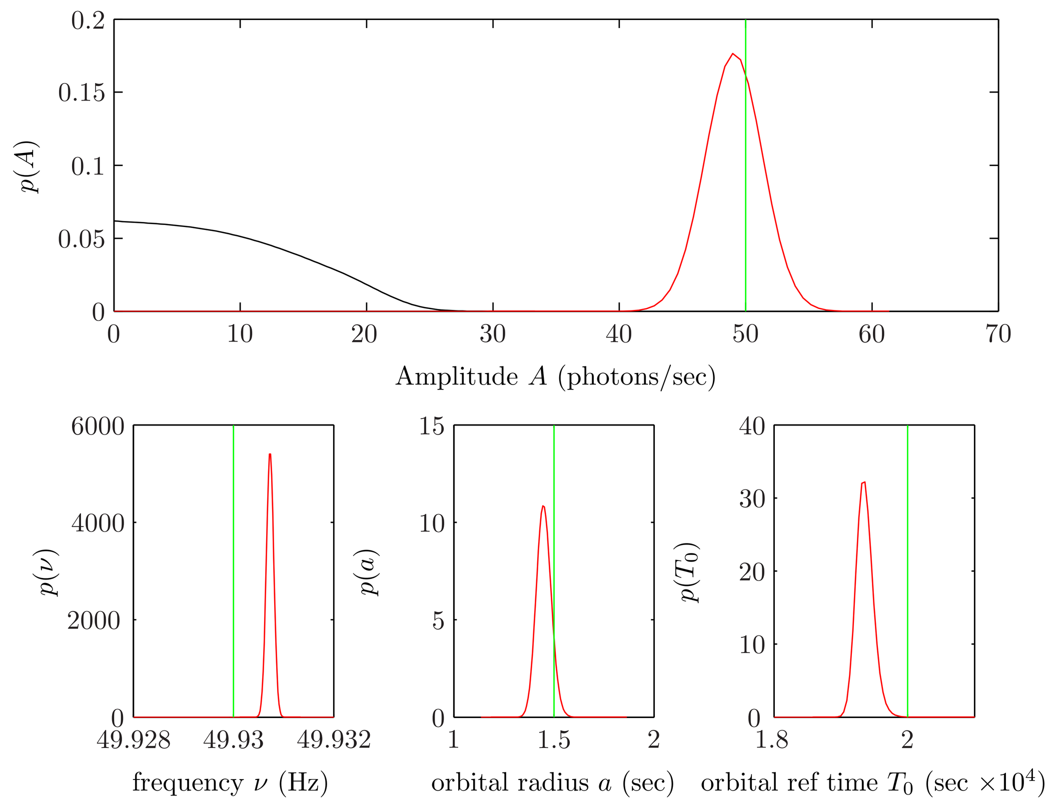

FIG. 2: The posterior PDFs on the signal amplitude, frequency, projected orbital radius, and orbital reference time for a simulated signal with Sco X-1 type parameters (orbital period $P = 68023$ sec and background count rate $r = 10^4$ counts/sec). The red curves represent the PDFs for the injected signal but also shown in the upper panel in black is the PDF for the signal amplitude for simulated data with no signal present. Vertical green lines indicate the true parameter values of the injected signal. This simulation was run on 101 short (~ 400 sec) data segments equivalent to $< 10\%$ of the total RXTE Sco X-1 observations and even on this reduced data set we obtain upper-limit sensitivities $\sim 0.2\%$ in fractional amplitude.

▸ Conclusions

We have described the basic framework for a Bayesian stack-slide search with particular application to RXTE X-ray archival data. We are at present in the process of performing this analysis on Sco X-1 data and have the intention to apply this search to the majority of the galactic LMXBs.

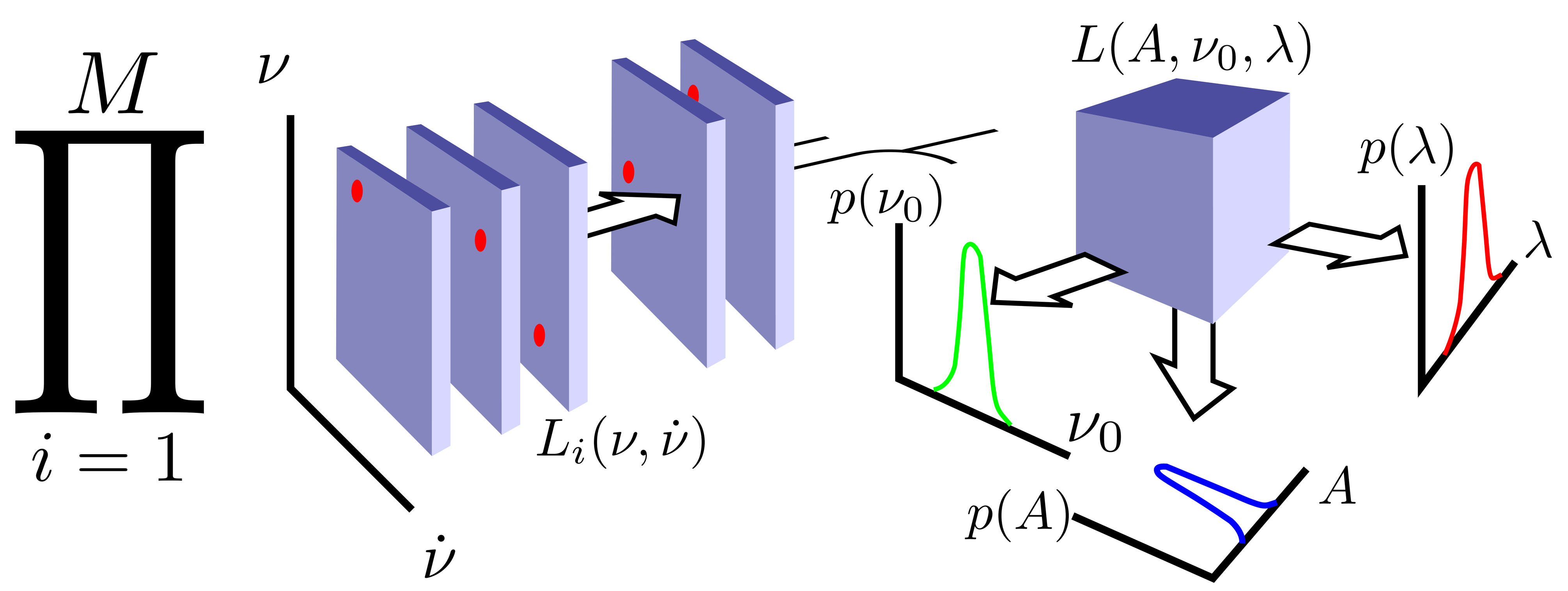

FIG. 1: A schematic representation of the search method. The thin blue slices on the left hand side represent the M segments of data for which the *phase-marginalised* likelihood has been calculated on a cubic grid in $\nu, \dot{\nu}$ space. The physical parameter space is defined by A, ν_0, λ where ν_0 represents the intrinsic pulse frequency and λ represents any additional search parameters. The red dots represent a possible $\nu, \dot{\nu}$ path taken by a signal with specific orbital parameter values. The likelihood values for each of these points are multiplied producing the likelihood as a function of the physical parameters. Posterior PDFs on these parameters are then computed by numerical marginalisation of this object represented by the three projected plots.